THE UNIVERSITY OF ALBERTA

RELATIONSHIP OF MATHEMATICAL RATIOS
TO VERBAL ANALOGIES

by

Ronald Keith Seward

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF EDUCATION

DEPARTMENT OF ELEMENTARY EDUCATION

EDMONTON, ALBERTA

APRIL 1966

ABSTRACT

The primary purpose of this study was to determine if exposure to the mathematical concept of ratio in grades five, six and seven affected students' success with tasks involving verbal analogies. Pairs of ratios, called proportional relations, may be expressed in the form, "a : b :: c : d". A similar form may be used to express verbal analogies (for example, horse : colt :: cow : calf). This similarity of expression suggests a relationship between mathematical ratios and verbal analogies.

A stratified random sampling of students in the grade seven classes of eight Edmonton schools was selected and separated into fifteen groups according to two criteria:

(1) intelligence, grouped as high, average or low according to scores on the Laycock Mental Abilities Test which had been administered when the students were beginning grade five.

(2) exposure to teaching of the ratio concept, grouped as follows: (a) ratio in grades five, six and seven, (b) ratio in grades six and seven, (c) ratio in grades five and six, (d) ratio in grade seven only, and (e) no formal instruction in ratio.

A verbal analogies test, the Verbal section of the <u>Academic Promise Test</u>, was administered by the classroom teachers and the data obtained submitted to an analysis of covariance.

Results of the analysis indicated a significant difference at the .05 level within the adjusted mean scores of the average intelligence groups. Further analysis using the Newman-Keuls method

indicated significant differences between the adjusted mean scores of two groups which had been exposed to the ratio concept and the group which had experienced no formal instruction in ratio. Students who had been exposed to the ratio concept in grades five, six and seven and in grade seven only scored significantly higher than the students who had not been exposed to the ratio concept.

Although there were no similar significant differences in adjusted mean scores when students of low and high intelligence were involved, studies of a non-statistical nature suggested that the study of ratio made it possible for the average student to perform almost as well on a verbal analogies test as the student of high intelligence. In contrast the average student who had experienced no formal instruction in ratio was able to perform little better on the same test than the student of low intelligence.

As a result of this study the implication might be suggested that exposure to the ratio concept has a positive effect on the grade seven student's ability to perform on a verbal analogies test. More carefully designed research of the topic should be attempted.

ACKNOWLEDGEMENTS

The writer of this thesis wishes to acknowledge his indebtedness to the chairman of the thesis committee, Dr. L. D. Nelson of the Department of Elementary Education, for his help in the selection of a topic and his persistent encouragement which made the completion of the project possible.

He would also like to express appreciation for the friendly cooperation of the principals and teachers in the schools visited, and of the supervisory staff of the Edmonton Public School Board.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION OF THE PROBLEM

The last decade has been marked by the publication of much educational literature of the central theme, "The Goals of Education." The various statements have been summed aptly in the National Education Association publication, The Central Purpose of American Education.

> The purpose which runs through and strengthens all other educational purposes - the common thread of education - is the development of the ability to think...To say that it is central is not to say that it is the sole purpose or in all circumstances the most important purpose, but that it must be a persuasive concern in the work of the school.[1]

Such statements have been translated into curriculum in many subject areas and in widely separated geographic locations. A subject area noted for its acceptance of this central theme is that of mathematics. The so-called modern mathematics programs attempt to teach the structure of the discipline. Through a knowledge of structure the student of mathematics, from a very early period of his school career, is expected to use the process of discovery (a thinking process) to gain mastery of the processes of mathematics. He is expected to reason out the steps in the processes of mathematics without the necessity of relying on rules or special tricks.

---

[1] National Education Association, Educational Policies Commission, The Central Purpose of American Education (Washington, D.C.: National Education Association, 1961), p. 12.

One area of the modern mathematics curriculum especially concerned with structure is that which utilizes the ratio concept to provide a model upon which to base the solution of many problem solving situations. The general structure is expressed in traditional terms as A : B :: C : D. The similarity of this structure to that of a type of item often present in intelligence tests and commonly called a verbal analogy, is apparent. In some tests it follows exactly the same format with words substituted for numbers (e.g., horse : foal :: cow : ___). In non-verbal tests, the same item might be presented through the use of pictures, again using the same format.

The similarity between the format which symbolizes the mathematical concept and that which symbolizes the ideas in the realm of language causes one to wonder if the teaching of the former has an effect on the child's mastery of the latter.

I. THE PROBLEM

The purpose of this thesis is to investigate the hypothesis that the study of ratio concepts in mathematics at the elementary school level influences the child's ability to make logical associations between non-mathematical groups, i.e., verbal analogies.

The specific null hypotheses to be tested are:

A. There is no difference in performance on a verbal analogies test between students who have had instruction in ratio in grades five, six and seven and those who have had no formal instruction in ratio.

B. There is no difference in performance on a verbal analogies test between students who have had instruction in ratio in grades six and seven and those who have had no formal instruction in ratio.

C. There is no difference in performance on a verbal analogies test between students who have had instruction in ratio in grades five and six and those who have had no formal instruction in ratio.

D. There is no difference in performance on a verbal analogies test between students who have had instruction in ratio in grade seven and those who have had no formal instruction in ratio.

II. SIGNIFICANCE OF THE PROBLEM

Since 1958 elementary school students in widely dispersed areas of Alberta have been making use of the Seeing Through Arithmetic texts as the basic tool for instruction in arithmetic.[2] In 1961 the texts were officially authorized for use in Alberta schools. The authors of the texts appeared to accept the hypothesis that the central purpose of the teaching of mathematics was the development of the ability to think.

> ...The program must give attention to integration in his learning so that he increasingly sees the inter-relationships among the things he studies in mathematics, and also sees the relationship of arithmetic to other branches of knowledge and to life outside school.[3]

The contents and organization of the program would tend to exemplify the theories of Bruner regarding structure:

---

[2]Maurice L. Hartung et al, Seeing Through Arithmetic (Grades three through six (Toronto: W. J. Gage Limited, 1961).

[3]Maurice L. Hartung et al, Charting the Course in Arithmetic (Chicago: Scott, Foresman and Company, 1960), p. 6.

> ...in order for a person to be able to recognize the applicability of an idea to a new situation and to broaden his learning thereby, he must have clearly in mind the general nature of the phenomenon with which he is dwelling.[4]

It has been suggested by Lindstedt that "many-to-many correspondence thinking transfers to making logical associations between non-mathematical groups and results in an increased competence in this area."[5] Lindstedt's terms "many-to-many correspondence" and "non-mathematical groups" refer to "ratio" and "analogies" respectively. Because Lindstedt's conclusions were based on a study involving elementary school students in Alberta, this seems an appropriate time to give more detailed attention to the suggested relationship.

III. EXPERIMENTAL SETTING

Since 1958 some grade five and six classes in the Edmonton Public School system have made use of the Seeing Through Arithmetic series of arithmetic texts. Children using these texts have been exposed to the ratio concept as a tool in solving many types of arithmetic problems. Other classes have made use of texts which have not introduced the ratio concept.

During the 1963 - 1964 term fifteen classes of grade seven students from the same system used the Seeing Through Mathematics

---

[4] J. S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1960), p. 12.

[5] Sidney A. Lindstedt, "Changes in Pattern of Thinking Produced by a Specific Problem-Solving Approach in Elementary Arithmetic" (unpublished Doctoral dissertation, University of Wisconsin, 1962), p. 103.

textbook,[6] Unit Four of which teaches "Conditions Involved in Rate Pairs." Seventeen other classes in the same schools used the traditional mathematics texts in which ratio was given little or no emphasis. The students in these thirty-two classrooms had graduated from numerous feeder schools and had been exposed to varying amounts of the teaching of the ratio concept during grades five and six. Therefore, it was possible to isolate students who have had varying amounts of formal instruction in ratio according to the following schedule:

i. Grades five, six and seven

ii. Grades six and seven

iii. Grades five and six

iv. Grade seven only

v. Little or no formal instruction in ratio.

It was also possible to obtain a rating of intelligence for each of these students which had been determined early in the grade five year. Thus, it was possible to further classify the students according to intelligence. The administration of a verbal analogies test completed the acquisition of data needed to test the hypotheses.

IV. DEFINITION OF TERMS USED

Ratio. A pair of numbers which express a relation. A study of the concept of ratio would involve an understanding of the use of

---

[6] Maurice L. Hartung, Henry Van Engen and others, Seeing Through Mathematics, Book 1 (Toronto: W. J. Gage Limited, 1962).

such number pairs to express the relation in mathematical terms. These pairs are sometimes referred to as rate pairs.

Proportional relation. An expression of the equivalence of two ratios (e.g. $2/3 \sim 4/6$ or $2 : 3 :: 4 : 6$).

Analogy. Given a pair of ideas between which a relation has been recognized, the application of this relation to a third idea in order to generate a fourth idea.

Verbal analogy. Any analogy which is expressed in words (e.g. A colt is to a horse as a cub is to a bear).

V. OUTLINE OF THE THESIS

The remainder of this thesis is organized into three chapters. In Chapter II the literature related to the manner in which children form concepts and manipulate them in order to reason and to solve mathematical problems is reviewed. A second section of the chapter outlines the methods prescribed for the teaching of the ratio concept and its use in problem solving. A final section includes a discussion of the literature regarding the reasoning process referred to by the term "analogy".

In Chapter III the design and procedures used in the experiment are outlined.

In the final chapter conclusions resulting from the experiment and implications drawn from the study are reported.

CHAPTER II

REVIEW OF RELATED LITERATURE

This chapter includes three sections. The first section is a review of the literature related to concept formation by children and the use of concepts in the reasoning and problem solving processes. The section is an attempt to provide a conceptual framework with which to justify the teaching of abstract concepts in the elementary school. The second section is a discussion of the literature which specifically suggests the feasibility of teaching ratio concepts in the elementary school. The section also includes an outline of the methods prescribed by the authors of _Seeing Through Arithmetic_ and _Seeing Through Mathematics_ for the teaching of the ratio concept and its use in problem solving situations. The final section is a review of the literature concerned with thinking patterns associated with analogies.

I. CONCEPT FORMATION, REASONING AND PROBLEM SOLVING

Much of the research into concept formation, reasoning and problem solving has involved college students and adults. The work of Piaget, who made careful records of observations of the mental growth of children, was ignored until recent times when the emphasis changed to research involving children. The works of Dienes, Van Engen and Isaacs have followed the work of Piaget and provide a framework which leads to the suggestion that the introduction of the

ratio concept at the elementary school level provided a powerful tool in developing reasoning and problem solving abilities.

What are concepts? Van Engen provided a summarizing statement which was an attempt to gather the general features covered by various available definitions. He chose the definition by Vinacke for this purpose:

> They must be regarded as selective mechanisms in the mental organization of the individual, tying together sensory impressions, thus aiding in the identification and classification of objects.[1]

The selective mechanisms might be regarded as sieves through which external stimuli must pass in order to arouse the proper responses. Sensory data are collected from varying sensory experiences and sorted according to common features. Once a set of sense impressions has been integrated to form a concept, future confrontation with the various external data will elicit a familiar response. It is suggested that primitive concepts (those developed through direct experience with the external world) act as the basis for developing very abstract concepts.

A point of conjecture among psychologists is whether or not a verbal or symbolic response is a necessary condition for the possession of a concept. Hebb, for one, believed that a verbal tag was not essential. Van Engen cited the fact that children often

---

[1] Henry Van Engen, "The Formation of Concepts," The Learning of Mathematics, Its Theory and Practice (Washington, D.C.: National Council of the Teachers of Mathematics, 1953), p. 82.

respond with the phrase, "it's just so" when pressed for an explanation to their solution to a problem as evidence of the existence of concepts on a subverbal level.

The attainment of concepts is facilitated by the abstraction (drawing out) of a quality from a variety of perceptual experiences. Gradually the parts of the percepts that are common are classified as a single quality to be recognized in future experiences. When the perceiver understands that he can use this common property to respond to a whole class of situations he has made a generalization. These stages, perception - abstraction - generalization, are thought to be the sequence of concept development in children.

Heidbreder[2] suggested an hierarchy of difficulty in the forming of different types of concepts according to the source of the concept. She found that the easiest concept formation resulted from reacting to drawings of pictured objects such as people, trees, et cetera. Abstractions made from reacting to drawings of forms such as circles or triangles were more difficult. Most difficult was the formation of concepts from abstractions obtained by reacting to facts about collections of objects.

She suggested that the manipulability of the thing to be conceptualized had a direct bearing on ease of conceptualization. Thus, for an adult, a drawing of an object such as a tree resembled

---

[2] E. Heidbreder, "The Attainment of Concepts: I. Terminology and Methodology," The Journal of General Psychology, 35 (1946), pp. 173 - 189.

the real object in such detail that the adult could mentally manipulate the various features of the object. As the object of perception became more and more abstract, previously gained concepts had to be relied upon to aid in the classification of sensory data.

The necessity of the use of manipulative materials in gaining concepts presented to Dienes the problem of what he chose to call "noise". From any one manipulative material, many concepts may arise. For example, a wooden block may illustrate the concept of "wood". Relevant concepts take precedence over others only to the extent to which the manipulator is able to discard sensory data which is irrelevant. If two types of situations can be seen as having similar features, noise is in the process of being discarded. Once noise has been controlled to a satisfactory level, a generalization is possible and time of symbolization is appropriate.

The matter of symbolization of concepts warrants some discussion. Dienes defined a symbol as "a sign that reminds us of something apart from the sign itself."[3] Symbols record the common properties of abstractions. There must always be some pattern of thought to be symbolized. In another article,[4] Dienes suggested that children, especially when they are forming primary concepts, need a bridge between the abstraction and the symbol. A sort of imagery must be developed to act as a bridge. The use of manipulative

---

[3] Z. P. Dienes, An Experimental Study of Mathematical Learning (London: Hutchinson and Company, Limited, 1963), p. 63.

[4] Z. P. Dienes, "On Learning of Mathematics," Arithmetic Teacher, 10 (March, 1963), pp. 115 - 126.

concrete materials will act as the best tool for building this imagery. Once the power of imagery is available the concept may be manipulated without the use of concrete objects and abstract structures may be developed.

Dienes further suggested that too early attempts at symbolization freezes the abstraction process in the stage at which the symbols are introduced. It is extremely important that a suitable power of imagery has been developed before a symbol is introduced if the children are to be expected to use symbols to control the manipulation of abstract structures.

Van Engen[5] discussed the syntactic and semantic dimensions of symbolization. By syntactic dimensions he referred to the use of a symbol in relation to other symbols in the sentence or equation. By semantic dimension, he referred to the fact that individual symbols represent objects, other symbols, simple events, or mental constructs. The referents (semantic dimension) must be clearly in mind when the child is using the symbols to represent mathematical situations. Once a child has illustrated his grasp of the semantic dimension by producing an appropriate mathematical sentence, the syntactic dimension of the symbols allows him to manipulate the symbols in such a way as to come to a solution which may then be referred back to the semantic dimension for translation back to the problem situation.

The possession of concepts in itself is of little value in the mental development of the child unless the concepts are used as

---

[5]Van Engen, op. cit.

a basis for further learning through manipulative or reasoning processes. To what extent can children be expected to make use of concepts to achieve further learning? Piaget's study of the develop ment of judgment and reasoning in the child was concerned with the structure of children's reasoning. The study shed light on the ability of children to make use of concepts by seeking out relations which may be used to build concepts of a more abstract nature.

Piaget described the structure of childhood reasoning according to stages of development. It is not the intention, here, to give an exhaustive account of Piaget's studies. A quotation followed by a brief explanation will serve to provide some background through which it will be possible to note that investigators of more modern vintage have made considerable use of Piaget's hypotheses.

> ...we can say that during the first stage the reasoning mind does no more than to 'imitate' reality as it is, without reaching any necessary implications; during the second stage the mind 'operates upon' reality, creating partly reversible experiments and thus reaching the consciousness of implication between certain affirmations and certain results; finally, in the third stage, these operations necessitate each other in the sense that the child realizes that by asserting such and such a thing he is committing himself to asserting such and such another thing. He has at last attained to necessary implication between the various operations as such, and to a complete reversibility of thought.[6]

Piaget suggested that stage one continues until the child reaches age seven or eight. It is characterized by the presence of much contradiction. The child believes that he can reason directly

---

[6] Jean Piaget, Judgment and Reasoning in the Child (New York: Harcourt, Brace and Company, 1928), p. 194.

about things without taking himself into account. His thought races from one particular to another without the sighting of relationships between the particulars. Thus, at one moment a stone can sink in water because it is heavy; at the next moment the stone can float in water because it is light. It cannot be expected that the child will be able to retrace the steps in his thinking; the relations used are not reciprocal and the thought is not reversible. Piaget gave the name "transductive thought" to this stage and refers to it as the period of intuitive thought.

Once there is a stock of concepts of objects and class, built up quickly during the sixth and seventh year and organized into coherent systems, logical operations evolve. Logical operations are really internalized responses which grow out of overt responses to environment. The child likes to perform mental experiments which operate upon reality. Premises are usually obtained from direct observation. He does not always see the need for testing his premises. He is able, however, to remake the mental experiences that he has already made in action. There is a shifting from action to thought. The thoughts are regulated by conscious judgments. Operations become reversible as the child realizes that relations must be reciprocal. (If I divide objects into four equal piles, I can recover the original whole by multiplying by four). Piaget called this stage the stage of concrete operations. It continues until the age of eleven or twelve.

When complete reversibility of thought is in evidence in the mental activities of the child, he is said to have reached the stage

of formal reasoning. Piaget outlined the condition which makes complete reversibility of operations possible:

> If a mental experiment is to be completely reversible, it is necessary to substitute for the objects offered in perception other more intellectual objects which shall be defined in just such a manner which will allow reversibility.[7]

At this stage the child can manipulate definitions or relations and by logical experiment he can make a choice suitable to the problem at hand. He realizes that by asserting a premise he commits himself to assert a logical conclusion. He realizes that the relation between concepts is often more important to the activities of thought than the concepts themselves.

A classic example which illustrates the stages of reasoning is that of the development of the understanding of the concept of left and right. If three objects are presented, the small child will be able to point to the object which is on the right and the one which is on the left of himself. At the earliest stages the child will learn to point to things which are to the left or right of himself. Later he will be able to pick left and right in relation to one another but when three objects are presented to him he will be unable to realize that the middle object could be classed as being to the right of one object and to the left of the other. At about age ten or eleven the child will have acquired sufficient relative notion of the relations of left and right without reference to specific objects to be able to rely on the relation remaining the same, whatever the point of view.

---

[7] Ibid., p. 192.

Van Engen[8] brought out the implications of Piaget's findings for the teaching of children. He suggested that, when initiating instruction designed to develop new concepts, attention must be focused on manual operations with the intention that the child will later be able to perform the same operations mentally. The child must have practice in visualizing actions. He must be able to visualize manual operations which have been indicated by words and to make generalizations about the operations. Eventually he will be expected to be able to ignore the kinds of objects and center his attention on the actions. Finally, he will be expected to manipulate concepts far removed from reality.

Dienes[9] suggested some implications regarding the learning of mathematics in the light of Piaget's findings. During the concrete operations stage the most effective learning of a permanent nature takes place as a result of concrete experiences. From these experiences a common property must be abstracted and a class formed. Abstract constructions may be built by means of primary abstractions already built. Before abstract constructions may be formed the child must have a variety of primary abstractions gained from a variety of concrete experiences.

Isaacs,[10] too, has written about the implications of Piaget's studies. He warned against the dangers of a false maturation

---

[8] Henry Van Engen, "Child's Introduction to Arithmetic Reasoning," School Science and Mathematics, 55 (May, 1953), pp. 358-363.

[9] Dienes, op. cit.

[10] J. Nathan Isaacs, The Growth of Understanding in the Young Child (London: The Educational Supply Association, 1961).

interpretation of Piaget's works. The order of succession is what matters, not chronological age, according to Isaacs. The process of development is not purely internal but each new level is only possible because of the experiences that have happened before. Through these experiences, a first framework is constructed and language mastered by the infant. The original model is gradually extended. The child's concrete activities fill in this framework in order that he will be capable of logical reasoning as he matures.

The work of Piaget and his followers strongly suggests that children must be exposed to experiences of a wide variety if they are to develop mature patterns of reasoning. In mathematics as well as all other areas of learning the progression from intuitive, to concrete, to formal reasoning must be fostered through the provision of such experiences as are necessary.

A child's ability to reason will be of little value unless the ability can be used in problem-solving situations. What does the literature have to say about the development of problem-solving ability as it relates to the field of mathematics? All too often the research has led to conclusions which do not get to the root of problem-solving difficulties. Judd may have isolated the real basis of the dilemma:

> Not infrequently the difficulty of learning arithmetic is increased by the fact that neither the textbook nor the teacher gives an adequate explanation of the way in which the abstract ideas which the pupils must learn to understand are to be abstracted from the problems or of the way in which concrete situations are to be reduced to purely mathematical formulas.[11]

---

[11] Charles H. Judd, Psychological Analysis of the Fundamentals of Arithmetic (Chicago: University of Chicago Press, 1927), p. 86.

In 1933, LaZerte[12] published the results of his studies in which he had endeavored to discover the mental processes of pupils when they attempted to solve problems in arithmetic. His subjects were children from various parts of Alberta who were in grades three to seven. An "envelope test" was devised to determine the pupils' ability to form correct judgments and to choose correct methods for solving problems when several alternative procedures were presented simultaneously. Series of envelopes containing descriptions of possible procedures forced the subjects to be concerned, first, with a generalization about method to be used, second, with activity related to reading and interpretation, and, finally, with a decision about method. The subjects were timed as they proceeded through each step and were allowed to return to the start of the test if they decided that they should.

The results of LaZerte's tests indicated that there appeared to be a gradual increase in analytical power related to the growth and age of the child rather than to the directed education of the child. Ability to use analysis in problem solving seemed to appear between grades four and six. LaZerte concluded that there was an hierarchy of thought habits in problem solving and that pupils experience a sudden increase in problem solving ability after basic habits and knowledge have been acquired. When the component abilities have been developed to a certain level a better organization of

---

[12] M. E. LaZerte, The Development of Problem Solving Ability in Arithmetic, A Summary of Investigations (Toronto: Clarke, Irwin and Company, Limited, 1933).

thought is possible. LaZerte suggested that little emphasis should be put on formal written solutions to problems until suitable organization of thought is possible.

The behavior characteristics of reflective thinking formed a basis for Hartung's[13] ideas about problem solving. He suggested that the following steps should apply to both reflective thinking and problem solving:

1) Recognize and formulate the problem.
2) Collect and organize the data.
3) Analyze and interpret the data.
4) Draw and verify conclusions.

Hartung believed that many of the textbook-problems did not give practice in developing the above characteristics. He suggested, however, that advances towards the improvement of problem solving techniques were taking place.

Finally, Lindstedt sought to identify patterns of thinking fostered by the Seeing Through Arithmetic program.[14] The authors of this series of elementary grade textbooks have attempted to structure the raw materials from which problems are made. The action within the problem situation is used as the relating structure, e.g. the concrete problems which might be represented by the equations, $25 + 17 = n$, $25 + n = 42$ and $n + 17 = 42$, all have the same structure because the action is additive. Lindstedt claimed that this approach

---

[13] Maurice L. Hartung, "Advances in the Teaching of Problem Solving," Arithmetic, 1948, G. T. Buswell, editor (Chicago: University of Chicago Press, 1948), pp. 44 - 53.

[14] Maurice L. Hartung and others, Seeing Through Arithmetic (Grades one to six) (Chicago: Scott Foresman and Company, 1957).

is justified if we are to believe as Piaget seemed to believe that the thought of the child is nearer to action than ours and consists of mentally pictured operations. The use of the equation makes it possible to guide the student from the stage of action to one of abstraction.

The subjects of Lindstedt's study were grade six students in city schools in Alberta. Children in the control group had received arithmetic instruction according to traditional methods which stressed the acquisition of computational skill and its application to the solution of problems. Children in the experimental group had used the *Seeing Through Arithmetic* texts in grades four, five and six. Lindstedt outlined problems which the experiment was designed to investigate. Two of the problems were:

A-1) Does the *Seeing Through Arithmetic* problem solving program produce any patterns of thinking that can be identified as related to this particular program, and which are different from those patterns of thinking promoted by a more traditional approach?

A-2) In what ways do pupils approach a problem which has a many-to-many correspondence situation?

To gain the necessary data, Lindstedt designed problem tests similar to the problems used by the authors of the texts. One set of problems substituted unfamiliar symbols for the regular numerals. After analysis of the data which had been gathered, Lindstedt concluded that:

The experimental group:

(a) related the mathematical model more closely to the action of the problem;

(b) was more cognizant of the referents to the symbols which were used;

(c) approached certain comparative problems by using many-to-many correspondence directly, rather than the rule-of-three approach. This resulted in a pattern of thinking that gave them more competence in solving very difficult problems of this structure and problems with imaginative settings and with unfamiliar words and number symbols.[15]

The study of the literature appears to suggest that children approaching the end of elementary school are able to make use of abstract concepts in problem solving situations where formal reasoning is required. The reporting of Lindstedt's study serves to lead to a description of the methods of teaching the ratio concept and its use in problem solving situations.

II. TEACHING RATIO CONCEPTS

As early as 1914, teachers of arithmetic were being encouraged to teach ratio. Brown and Coffman[16] suggested that the fostering of the "Rule of Three" method should be discarded in favor of the teaching of the ratio concept.

According to the "Rule of Three" method, students were taught to follow three steps:

1) state the relationship (e.g., five bananas cost 28¢);

2) reduce this by division, to a relationship involving unity, (e.g., one banana costs 28 divided by five, or five and three-fifth cents);

---

[15] Lindstedt, op. cit.

[16] Joseph C. Brown and Lotus D. Coffman, How to Teach Arithmetic (Chicago: Row, Peterson and Company, 1914).

3) Use this unity relationship to solve the problem by multiplication (e.g., 15 bananas cost fifteen times five and three-fifths, or 84¢).

Brown and Coffman recommended the use of ratio because it would foster a reasoning process rather than a mere mechanical procedure in securing answers to problems. The ratio concept can be utilized through the use of an equation called a proportion which is actually a comparison of ratios which states the equality of two equal ratios. The fundamental principle of a proportion is that the product of the means is equal to the product of the extremes.

Although the use of ratio as a powerful tool in problem solving seems to have been recognized for many years, the custom of linking ratio with fractions has made it difficult to make use of ratio until the junior high school grades. Although not explained as such, the "Rule of Three" method has been used at the grade six level. One of the presently authorized texts in Alberta, Study Arithmetic, suggests this method of attack under the heading "Thinking About Fractions in Three Ways". The following problem will serve to illustrate:

> 7. The twelve planes made by the other boys in the school were three-quarters of all the planes. How many planes were there in all?
>
> To answer this question, you should find four-fourths, or all, of the planes. Three-fourths is twelve planes; so one-fourth of the planes is twelve divided by three, or four planes. Four-fourths, or all of the planes is four times four or sixteen planes. Another way to find this answer is to divide twelve by three-quarters.[17]

---

[17] F. B. Knight, J. W. Studebaker, and G. M. Ruch, Study Arithmetic, Book Six (Toronto: W. J. Gage and Company, Limited), p. 262.

A similar exercise "Thinking About Decimals in Three Ways" is included in the text.

The ratio concept is introduced in the second chapter of the grade seven text, Winston Mathematics, Intermediate Book One.[18] The chapter is headed "How Well Do You Understand Fractions?" Two pages are devoted to the subject. Under the heading "Ways to Compare Numbers" the text gives a definition of ratio; "Ratio means how many times another number one number is."[19] It is further stated that the ratio can be read as "two to three" or "two-thirds" for example. On a later page pupils are shown how to express ratio as a decimal and a percentage. In all cases the ratio is manipulated as if it were a fraction.

Until the ratio concept was considered to be worthy of being taught as a concept rather than a manipulative tool it had little consideration in mathematics textbooks. Trimble[20], although he considered work with ratio to be properly placed in junior high school textbooks, recognized that the establishment of an elemental basis for the understanding of ratio should be the concern of teachers at all levels. In recognition of the theory that the learning of concepts must be grounded in muscular activity, he suggested that it was easier for young children to consider number pairs such as 2/3

---

[18] W. L. Stein, et al, Winston Mathematics, Intermediate Book One (Toronto: John C. Winston Company Limited, 1952).

[19] Ibid., p. 68.

[20] H. C. Trimble, "Fractions Are Ratios Too," Elementary School Journal, XLIX (January, 1949), pp. 285 - 291.

as a ratio than as a fraction. The pair could be interpreted as "for each three take two". In a similar manner much of the confusion in the language of per cent could be avoided. Rate problems, too, may be easily illustrated; "for each three of one sort take two of another". Trimble suggested to teachers that the unified treatment of the ratio concept of a fraction would act to supplement the other fraction concepts.

Hartung, Van Engen and Gibb have played a major role in giving the ratio concept a place in the elementary school curriculum. They have done so by carefully delineating the ratio concept as compared to the fraction concept. Hartung[21] recognized that the ratio concept was basic to the understanding of the fraction concept. This theory was in direct opposition to the usual listing of "ratio meaning" as one of the meanings of "fractions". He stated that the family name of the ratio concept is "relation" and that one of the simplest of mathematical relations is that between pairs of numbers often called "ratio". He used the term "many-to-many correspondence" when he discussed the ratio concept in order to link it to the more fundamental one-to-one correspondence.

Van Engen and Gibb[22] clearly explained the difference between fractions and ratios. Fraction number pairs may be manipulated like natural numbers and can, therefore, be placed in the realm of

---

[21] Maurice L. Hartung, "Fractions and Related Symbolism in Elementary School Instruction," Elementary School Journal, LVIII (April, 1957), pp. 377 - 384.

[22] Henry Van Engen and E. Glenadine Gibb, "Structuring of Arithmetic," National Council of Teachers of Mathematics, Twenty-fifth Yearbook (Washington: 1960), pp. 31 - 61.

numbers, whereas ratios (the symbols for rate pairs), do not fit so readily. They suggested that fractions must be introduced to the student by measurement situations whereas the ratio concept must be introduced through the study of rate or comparison situations.

In another article Van Engen[23] discussed rate pairs and fractions. He defined a rate pair as a pair of natural numbers representing a many-to-many correspondence. He warned that rate pairs should not be called natural numbers because they are not subject to the operations to which numbers may be submitted. However, rate pairs and fractions do have common properties:

1) If they belong to the same set (are equivalent) then their cross products are equal.
2) If each component of the pair is multiplied by a natural number another member of the same rate pair (or equivalent fraction) is obtained.
3) Components of rate pairs and fractions may be divided by a common factor.
4) Any given rate pair belongs to one and only one set of rate pairs.

Van Engen drew the curricular implication that rate pairs should be taught before fractions because the physical situation of rate pairs is simpler to abstract ideas from. The basic properties of fractions may be developed from rate pair experiences. Finally he suggested the feasibility of introducing these ideas at the grade four level.

The authors of the Seeing Through Arithmetic and Seeing Through Mathematics textbooks have given the ratio concept a significant

---

[23] Henry Van Engen, "Rate Pairs, Fractions and Rational Numbers," Arithmetic Teacher, 7(December, 1960), pp. 389 - 399.

place in their programs. A brief non-technical view of the principles and points of view behind the program at the elementary school level is given in Charting the Course in Arithmetic.[24] The student text[25] and teaching guide[26] were designed to help the student build a structure on which to base problem solving situations. The authors recognized that many problems which frequently occur in modern society involve the ideas of rate and comparison. The idea of rate is involved when one quantity is paired with another (e.g., speed of fifty miles per hour). Comparisons involve two groups of objects and two numerals are required to express the comparison (e.g. Bill has ten toy cars and Frank has seven toy cars). To express either of these situations a numerical relation can be used. The pair of numerals which are used is called a ratio. To make use of the ratio concept in problem solving, a comparison of ratios must be employed. Two equivalent ratios may be compared. This comparison may be expressed as an equation (e.g., 7/10 = 21/30), which is commonly referred to as a "proportional relation". A proportional relation is essentially a comparison of the results of two previous comparisons. Through its use many problems may be structured so that they may be easily solved.

---

[24]Maurice L. Hartung et al, Charting the Course in Arithmetic (Chicago: Scott, Foresman and Company, 1960).

[25]Maurice L. Hartung et al, Seeing Through Arithmetic (Grades Three Through Six) Toronto: W. J. Gage, Limited, 1961).

[26]Maurice L. Hartung et al, Teachers' Guide, Seeing Through Arithmetic (Grades Three Through Six) (Toronto: W. J. Gage, Limited, 1961).

The building of the structure which will form the foundation for the ratio concept begins at grade one level of the Seeing Through Arithmetic program.[27] Through manipulation of concrete materials the child comes to understand the concept of one-to-one correspondence. This concept is considered to be one of the "few simple ideas which provide the foundation stones upon which all arithmetic is built."[28] The idea of one-to-one correspondence is used when individual members of one group of objects are paired with members of another group of objects. A familiar situation would be that of pairing chairs with children or dolls with doll buggies. No matter how different they may be in other aspects, groups of objects are the same in number if they can be paired in one-to-one correspondence.

In grade two the child is expected to generalize the idea of correspondence to include correspondence such as two to three or one to ten between similar objects and objects which are quite different. Three pages of the grade two text, Numbers in Action,[29] are devoted to this task. Pupils are expected to see that, in a collection of toys, two balls out of every three are red and one out of every three is blue. From this it follows that there are two red balls for each blue ball. In a similar manner, for each twenty red balls there

---

[27] Maurice L. Hartung, et al, Numbers We See (Toronto: W. J. Gage, Limited, 1955).

[28] Maurice L. Hartung, et al, Charting the Course in Arithmetic, op. cit., p. 7.

[29] Maurice L. Hartung, et al, Numbers in Action (Toronto: W. J. Gage, Limited) pp. 86 - 88.

would be ten blue balls. Correspondence may be set up between objects that are quite different (e.g., boys and airplanes or dolls and blankets). Many different situations are explored with the concept of many-to-many correspondence the common factor.

With an understanding of elemental concepts established in the primary grades, the grade five student is expected to use pairs of numerals (called ratios) to express number relations. Charting the Course in Arithmetic suggests that "the idea of 'relation' is of great mathematical significance."[30] It is not expected that pupils at the elementary level should understand the definition of the concept. Situations which involve rates and comparisons are common in arithmetic problems and are rather simple relations. However, an effective symbolism must be presented to the pupils. They become familiar with the symbolism of ratio through concrete and pictured rate and comparison situations. Many objects which children need to buy (candies, pencils, et cetera) are sold at rates which involve many-to-many correspondence situations (three for ten cents). Such experiences form the basis of concrete situations for the arithmetic lesson. Consideration of a rate of twelve per fifteen and the possible reordering of the objects to illustrate rates of four per five or sixteen per twenty lead the child to understand that different ratios can express the same rate. Care is taken that the terms "rate" and "ratio" are not confused. Rate is used to describe

---

[30] Hartung et al, Charting the Course in Arithmetic, op. cit., p. 103.

the relationship (so many items for so much money) while ratio is used as the name of the pair of numerals which describe the rate.

Ratios are linked to rate and comparison situations until the child is able to think of ratio in a purely abstract form. A further step is taken when the pupil learns to convert one ratio to an equivalent form. The next step is to apply this knowledge of ratio to problem solving situations. The concept of proportional relation becomes involved. Of the four numerals of a proposed proportional relation, three are known and one is unknown. It is not important which numeral becomes the first term of the original ratio. But once the first term is selected, the student must be consistent when he selects the first term of the equivalent ratio. It is at this point that he must keep the relationships clearly in mind. He must be able to make conscious, logical judgments.

In the early stages of using ratio to solve rate and comparison problems Seeing Through Arithmetic, Grade Five uses a procedure for which a full explanation is provided. To facilitate explanation of the procedure the following problem will be used: "John earns 25¢ a day, and Bob earns 15¢ a day. When John has earned $1.25, how much will Bob have earned?" It is the task of the student to set up a proportional relation in which one term will be unknown. The ratio 25/15 represents the relation between the two boys' daily earnings. Another ratio representing the earnings for a longer period of time must be established. Since $1.25 is the amount of earnings coinciding with the 25¢ daily earnings, 125 must be the first term of the second ratio. The second ratio must be 125/n,

"n" taking the place of the unknown term. The proportional relation will then be 25/15 equals 125/n. To solve the problem a multiplier must be found which will convert 25 to 125. The multiplier is, of course, 5. Then 25 times 5 is 125 and 15 times 5 is 75, so the new ratio is 125/75. This shows that n equals 75. This method eliminates the necessity of short cuts and makes it possible to introduce an unlimited number of problems which grade five children can solve with their limited knowledge of operations with fraction numerals. (If the above problem were to be solved using the "rule of three" method, the ratio 25/15 would have to be converted to 1 and 2/3 and 125 would have to be divided by this mixed number.)

In the early stages of problem solving by ratio the child is expected to decide by a process of examination how to convert the original ratio to its equivalent, thus finding the unknown term. In grade six the process, named the "ratio test" is introduced. The child learns that two ratios are equivalent if the products determined by multiplication of the extreme terms are equal, i.e., "a/b is equivalent to c/d if and only if ad equals bc". No attempt to show proof of the test is considered at the grade six level.

It should be noted at this point that each step in the teaching of the ratio concept and the method of using ratio to solve arithmetic problems is based on the properties of ratio and that no devices or short cuts which will later have to be unlearned are introduced.

A wide variety of problems may be expressed in terms of ratio. In the <u>Seeing Through Arithmetic</u> program the technique is

used to solve problems involving area, conversion of measures, and percentages as well as the basic rate and comparison problems. At the completion of grade six, the carefully taught student has had ample opportunity to make use of the ratio concept in abstract as well as concrete situations.

Unit Four of the grade seven program in mathematics, Seeing Through Mathematics, Book One attempts to "extend the work with conditions in two variables to include conditions that involve rate pairs."[31] It is an attempt to use strict mathematical notation to express the ratio concept. To develop a definition of proportional relation, equivalent ordered pairs are defined as follows: (a,b) and (c,d) are equivalent if and only if "ad" equals "bc". Thus, a proportional relation may be defined as a set of ordered pairs in which each member is equivalent to each of the other members. These ideas are introduced by consideration of mathematical symbols with some attempt to relate physical situations to the mathematical ideas. The technique might be considered to be telescoped version of the Seeing Through Arithmetic approach with the addition of appropriate symbolism.

Although it is not the intention here to provide a thorough explanation of the teaching of the mathematical ideas and symbolization which is taught in the first three units of the grade seven text, some explanation must be offered in order to explain how the ratio concept fits into the idea of conditions in two variables.

---

[31] Henry Van Engen et al, Teaching Guide, Seeing Through Mathematics, Book One (Chicago: Scott, Foresman and Company, 1963), p. 121.

The understanding of the meaning of some terms is necessary if the reader is to follow the explanation. First, the mathematical term, sentence, may be thought of in much the same way as the same term used in the teaching of language. A sentence which includes a placeholder is considered to be open. When the placeholder is replaced by a numeral, the sentence is considered to be closed. A condition is explained as being the requirement expressed by an open sentence. For example, x plus 1 equals 5 is an open sentence involving the requirement of equality. The condition must be thought of as the idea behind the sentence and the variable is the idea behind the placeholder. Conditions in two variables are conditions which may be expressed using open sentences which have two placeholders. Compound conditions are composed of two or more simple conditions joined by logical connectives such as "and" or "or".

The distinction between "equality" and "equivalence" should be noted:

> Notice that "Equivalence" and "Equality" are not used in the same way. You know that (2,7) is equivalent to (4,14) because 2 times 14 equals 4 times 7. However (2,7) is not equal to (4,14) because (2,7) and (4,14) are not the same ordered pairs. They are equivalent ordered pairs.[32]

The authors of the text have made use of non-mathematical sentences, such as "m is the capital city of n", to illustrate the ideas of ordered pairs and conditions in two variables. They state that

---

[32] Henry Van Engen et al, Seeing Through Mathematics, Book One, Part One (Toronto: W. J. Gage, Limited, 1962), p. 169.

"ordered pairs of objects are fundamental to a great deal of modern mathematics."[33] The teaching of rate pairs is only the first of the ideas for which sets of ordered pairs are basic.

A simple rate situation, i.e., 3 pencils per 12 cents, is used to indicate that equivalent ordered pairs constitute proportional relations. In the first lesson students are taught to use the definition of equivalent ordered pairs to decide if the ordered pairs are equivalent. (In grade six this procedure was called the "ratio test".) In the second lesson "rate pair" and "ratio" are introduced. As usual the authors are careful to distinguish between the idea, i.e., rate pair, and the name for the idea, i.e., ratio. The symbolism for expressing a proportional relation is illustrated by $1/15 \sim 2/30$ and is read "one to fifteen is equivalent to two to thirty". The third lesson demonstrates how the definition of equivalent ordered pairs may be used to determine missing components of proportional relations. A set of equivalent ordered pairs with one variable is symbolized, i.e., $16/8 \sim 4/x$, and the definition is used to transpose the symbols to the form, "16x equals 32", an equation which is easily solved for x.

It remains to teach the student how to use rate pairs to solve rate and comparison problems. A simple condition in one variable such as is illustrated in the previous paragraph may be used. This condition, according to the authors of the text, does not involve a set of ordered pairs. The use of a compound condition in two variables will require a universe of ordered pairs. The term "universe"

---

[33] Ibid., p. 103.

refers to the "set consisting of all the objects that can be used as replacements for the variable in the condition."[34] The condition, $6/8 \sim x/y \wedge x$ equals 24, which may be read, "six to eight is equivalent to x to y and x equals 24" is a compound condition related to the following problem:

> Jerry and Tim compared their walking speeds. They found that Jerry walks 6 blocks while Tim walks 8 blocks. When Jerry has walked 24 blocks, how many blocks will Tim have walked?[35]

By using the compound condition the student is reminded of the full problem situation. It is necessary that he keep in mind the referents for each of the place holders and the order in which they must be placed if the ordered pairs are to be equivalent in relation to the problem situation.

The remainder of Unit Four of the text introduces the student to problems involving per cent as rate pairs, problems involving compound conditions about rate pairs, and abstract problems involving rate pairs. This last set of exercises gives the student practice in using his understanding of the ratio concept in strictly abstract situations (e.g., "Certain rate pairs are equivalent to 3/7. The first component of each of these rate pairs is less than 25. What are the rate pairs?").[36]

It should be expected that students, if taught carefully to the level expected in grade seven, will have a sound understanding

---

[34] _Ibid._, p. 16.

[35] _Ibid._, p. 180.

[36] _Ibid._, p. 200.

of the number relation which may be expressed by a pair of ratios. The section which follows is a review of the literature which pertains to a type of verbal relation which may be expressed by a verbal analogy.

III. ANALOGIES

The subject of analogies may be approached from two aspects, that of the psychologist and that of the philosopher. Psychologists such as Burt, Spearman and Raven were interested in analogies as a tool of measuring mental capacity. Peel included a discussion of analogy in his account of the growth of children's thinking, while Watts and Upton were interested in analogy as it relates to language. Philosophers Aristotle, St. Thomas Aquinas, Cardinal Cajetan and Wittgenstein were interested in general theories of analogy. This section attempts to discuss analogy as it interested a psychologist. Because the writer does not pretend to be knowledgeable with respect to philosophical aspects of the subject only a brief reference to the subject is included.

As early as 1911, Burt set out to elaborate tests designed to measure higher mental processes:

> By "higher" mental process I understand one which is dependent on and directly or indirectly involves other mental processes of a relatively simpler kind. The highest mental processes of all are those classes together under the term "reasoning".[37]

---

[37]Cyril Burt, "Experimental Tests of Higher Mental Processes and their Relation to General Intelligence," Journal of Experimental Pedagogy, Vol. I, No. 2 (November, 1911), p. 95.

After giving a series of carefully planned tests to children from ages eleven and one-half to thirteen and one-half he distinguished between a mechanical association termed "association by contiguity" and "association by similars". Introspection of some of the children taking the test which involved solutions of questions, association of words, speed of writing and completion of sense indicated that the process used in supplying the answers was one of mechanical memory rather than reasoned reflection. This is "association by contiguity."

One of Burt's tests to determine ability to make "associations by similars" was a test which required completion of analogies. The test, it is claimed, involved "the perception, implicit or explicit, of a relation and the reconstruction of an analogous one by so-called relative suggestion".[38] Burt is referring to the common verbal analogies test. The form used by Burt consisted of one hundred examples illustrated by the following:

1. Eating: Drinking: Hungry: ____________?

He described the test as a series of problems in proportion or "rule of three", with concrete ideas substituted for numerals. Burt found that the reliability coefficient of the test and the correlation with intelligence were unusually high, .92 and .52 respectively. Burt concluded that the analogies test involves essential elements of logical inference. In likening the verbal analogies test to "rule of three" problems he implied a relationship between the type of thinking involving ratio and analogy.

---

[38]*Ibid.*, p. 100.

Spearman's The Abilities of Man[39] was primarily concerned with a person's ability to cognize. He suggested that the process of analogy is an important aspect of the thinking process.

> But by analogy with his own inner experience a person proceeds to generate thoughts--and even percepts--of other persons around him. The form of such mental generation is laid bare in the mental tests of analogies...[40]

He mapped out the domain of cognition with three quantitative laws under which all new cognition may be placed. Briefly the laws may be stated as follows:

1) A person has power to observe what goes on in his own mind.
2) When a person has in mind any two or more ideas he has the power to bring to mind a relation between them. This is the eduction of relations.
3) When a person has in mind any idea together with a relation he can bring to mind the co-relative idea. This is eduction by correlates.

It was suggested by Spearman that the third of these neogenetic laws includes the first two.

Spearman sub-divided the relations into ten categories, one of which he terms psychological. Psychological relations arise from the dual constitution of mental processes; a subject and an object are implied. At first thought psychological relations, according to Spearman, might have been placed under the first law. On closer observation, he suggested, they should be classified under the third law thus giving them a place in the higher structure of cognition.

---

[39] C. Spearman, The Abilities of Man (London: Macmillan and Company, 1932).

[40] Ibid., p. 180.

The following quotation clearly indicates that Spearman considered the process involved in solving the mental tests of analogies to be based on the psychological relation which may be classified under the third law.

> First a pair of ideas is given, between which a relation has been cognized; and then this relation has to be applied to a third idea, so as to generate a fourth one called a correlate.[41]

If we think the "ideas" and "relations" as mathematical in nature the above model fits the idea of ratio. Spearman, like Burt before him, linked ratio and analogy.

Raven has developed the "Progressive Matrices" tests[42] which were designed to assess the chief cognitive processes. He concluded that the majority of adults rely completely on inference by analogy as a consistent method of reasoning. He suggested that young children seldom show this degree of intellectual sophistication. He suggested at least five qualitative developments in the order of intellectual activity, the first step being the ability to distinguish identical figures from different figures and later dissimilar figures, and the last being the ability to compare analogous changes in the characters perceived and adopt this to a logical method of reasoning.

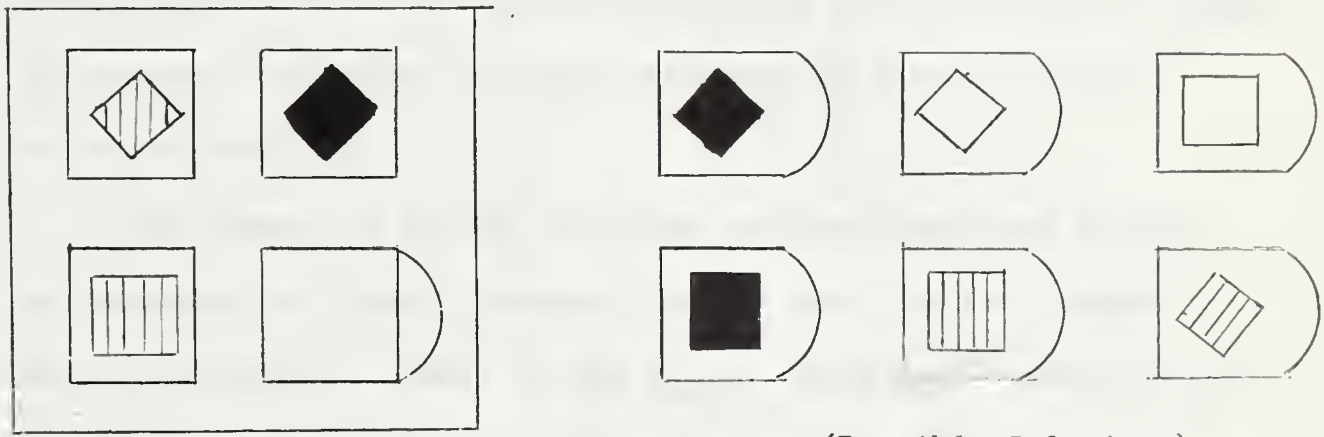
The matrices consist of designs which are printed on boards with one part of the design missing and six designs from which one must be chosen to complete the design. Tests are also available in booklet form. Two examples are shown to illustrate the tests in Figure 1.

---

[41]Ibid.

[4]]J. C. Raven, Progressive Matrices (London: H. K. Lewis and Company, 1956).

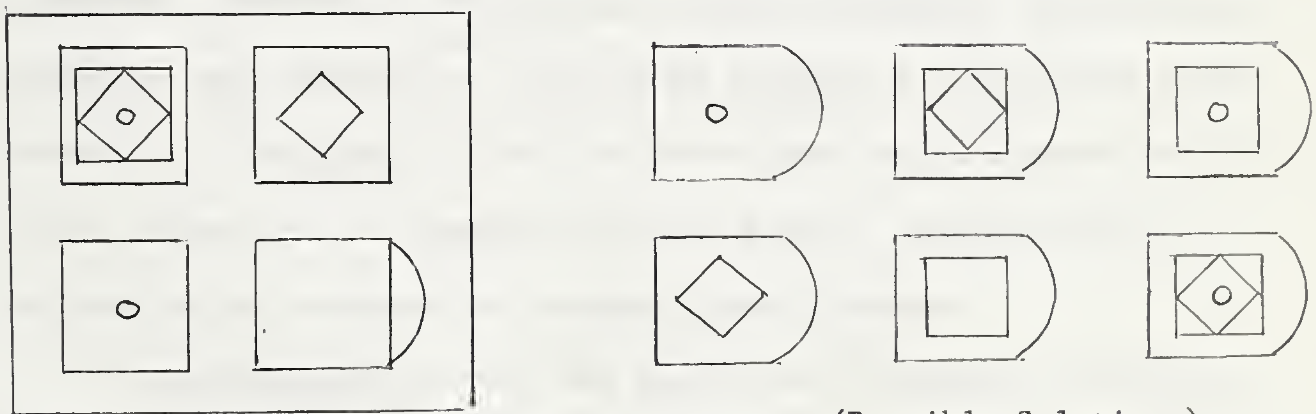
B9 - Concrete or Coherent Reasoning by Analogy to Assymetrical Change in Modified Figure



(Possible Solutions)

(Missing Item)

B12 - Discrete or Abstract Reasoning by Analogy to the Logical Double Subtraction of Given Characters from a Given Figure



(Possible Solutions)

(Missing Item)

Figure 1. Example test items from Ravens Progressive Matrices.

The arrangement of the component parts of the test figures is somewhat similar to the arrangement of a mathematical ratio.

Experimental data gathered by Raven suggested that by the time the child is over the age of nine he is usually able to reason by analogy and to adopt analogy as a consistent method of reasoning. Raven suggested that subsequent educational progress seems to depend largely upon the degree to which the child is able to use this method of thinking.

The growth of pupils' thinking has been described by such psychologists as Piaget, Inhelder, Wertheimer, Duncker, Bruner, Isaacs and Skinner. Peel, in The Pupils' Thinking,[43] added to the ideas of these men by reporting some original work carried out at Birmingham. He suggested that the basic mechanism of thought is relation finding or association and considers four kinds of thinking classified according to content and results--thematic, explanatory, productive and integrative. Each kind includes all the kinds which precede it. According to Peel the first three kinds account for most of the content of the thought of school pupils. Analogy would be included in the integrative thinking classification.

Also suggested by Peel are four types of logical structures which are revealed in childhood thinking:

1) simple classification;
2) ordering members of a class;
3) classification according to two categories at a time;

---

[43]E. A. Peel, The Pupils' Thinking (London: Oldbourne Book Company, Limited, 1960).

4) a one-to-one pairing of partners from two ordered classes according to properties which characterize the seriation within each class.

He suggested that analogy is the simplest type of this last logical structure. Consider the analogy, "mare" is to "foal" as "cow" is to "_____". "Mare:foal" is a class of two ordered elements; "cow:_____" is another. To complete the analogy it is necessary to take "mare" and "cow" as an ordered class and then generate another class.

Peel agreed with Piaget that children below a certain age (probably age twelve) are not able to depart from material relations or to consider or test possible hypotheses. About the age of twelve there seems to be a marked change. Through the use of words they seem to be able to internalize actions and form theories which can be tested.

Of some concern to Peel were the various ages suggested by different researchers concerning the time when the change from concrete thinking to formal thought occurs. He suggested two explanations. First, children may use concrete thinking even though they are capable of formal thought. He also suggested that if the concept of mental age is used the phases of development can be placed more firmly. Peel suggested that the few studies which have been conducted indicate that teachers might be able to hurry the progress toward the formal stage of thinking by providing added experiences combined with comments, suggestions and criticisms. However, too wide a gap between instruction and the thinking level of the children must be avoided. Only a mechanical skill, which leads to the masking of the thinking powers, is the result of such a gap.

One might infer from the work of Peel that children would achieve success with analogies of a very concrete nature before they reach the mental age of eleven or twelve. Only after they are able to reason formally would they be able to solve the more abstract verbal analogies.

The part played by language in mental development must be considered relevant to the present discussion. Watts designed objective tests to attempt to outline the principal stages through which children normally must pass in their effort to say and write what they think and feel. He hypothesized an orderly sequence in development which is the same for all children.

In Chapter Eight of The Language and Mental Development of Children,[44] Watts discussed the importance of metaphor and analogy in language. Because language as an instrument of classification is imperfect for classifying the most intangible things of life, it is necessary to use analogy as an aid in comparing these intangible things with familiar things. He suggested that when first used analogies may have been the result of flashes of original insight into the nature of human behavior. In ancient Greece metaphor and analogy were much used. It was first used in mathematics.

Watts classified metaphorical expressions into four levels:

1) naive identifications which children use (e.g., goldfish);

---

[44] A. F. Watts, The Language and Mental Development of Children (London: George C. Harrop and Company, Limited, 1944).

2) descriptive identification (e.g., wings of a building as if identical to wings of a bird);

3) analogy proper, a set of concrete circumstances thought of as though identical with another set;

4) proverbial experience for conveying fundamental truths.

He suggested that teachers teach by analogy too often. Children below junior high school age rely on one type of association which occurs and recurs. The immature child is not able to withstand the suggestion that the word which has the strongest association in his mind is not necessarily the correct choice. For instance, consider the verbal analogy, up:down::ceiling:(roof, high, walls, floor). If "roof" has the most powerful association with "ceiling" the child is very likely to choose that response to be the correct one. This suggests, as Peel did, that children are not able to consider and test possible hypotheses. The association of logical opposites appears to be most easily recognized by children but gradually, better classifications emerge, i.e., genus, species, part, whole, et cetera. Watts suggested that some bright ten year olds have started working out such classifications.

Watts relied on Piagetian ideas when he speculated as to how immature children begin to reason. He suggested that they often use analogy to think closely about things but that they jump from one idea to another by a kind of intuition of resemblance. It is purely transductive reasoning. He suggested that to use analogy safely children must be able to employ safeguards and checks, and warns that they need careful guidance past the pitfalls.

> The value of an analogy is that it suggests the existence of parallels where we have not perhaps suspected them; they provide us with an hypothesis which must however be tested farther before we can safely draw any conclusion.[45]

Watts suggested that a real respect for correct reasoning is needed if fruitful use is to be made of analogous thinking. He suggested that such respect can be engendered through study of mathematics.

Upton has produced a series of exercises designed to develop the ability to use words in problem solving. Section Five of *Creative Analysis*[46] consists of textual sections which explain principles and exercises which provide problems involving analogy. The exercises are based on theories outlined in *Design for Thinking*[47] which discusses the part language plays in human life.

Upton stressed the importance of metaphor in language. He called metaphor the "hallmark of linguistic maturity"[48] and explained that the word is originally Greek but may be translated into the Latin word meaning "transfer". He used the symbolization of ratio in describing metaphor.

> In the proportional metaphor some thing "a" is seen to be in some meaningful relation to some thing "b" that has a meaningful resemblance to the relationship between some thing "c" and some thing "d".[49]

---

[45] *Ibid*., p. 209.

[46] Albert Upton and Richard W. Samson, *Creative Analysis* (Whittier: Whittier College Press, 1961).

[47] Albert Upton, *Design for Thinking* (Stanford: Stanford University Press, 1961).

[48] *Ibid*., p. 72.

[49] *Ibid*., p. 76.

He referred to the recognition of the symbolization of "a", "b", "c" and "d" as the "fundamental analogical act that characterizes the procedure of creative intelligence,"[50] and suggested that repeated experience in analogical thinking develops the power to get meaning which will lead to the solution of problems.

In Creative Analysis analogies are classified according to Figure 2. They may be symbolic or nonsymbolic. An analogy is classed as symbolic because it is made up of symbols which express proportional relationships which exist in nature, e.g. "The Lord is my Shepherd" is symbolic, the actual similarity between a shepherd and his sheep and God and the psalmist is nonsymbolic.

Upton has assigned terms to be used in discussing analogies. Analogies are said to inhabit universes of discourse. The analogical universe is the universe from which the familiar relationships and terms are borrowed. The contextual universe is the universe to which relationships and terminology are transferred. The meta-universe is the universe which includes the other two.

Upton used the term "artificial" to describe many verbal analogies because they do not occur naturally in our language but have been contrived. Naturally-occurring analogies, according to Upton, function in a practical way in literature, scientific discovery and other areas of creative thought.

Upton included metaphor as the last and most important relation which may prompt the mind to give new sense to old words and suggests that systematic training in interpretation and formation of metaphors has been the only factor that could account for marked increase in intelligence quotient.

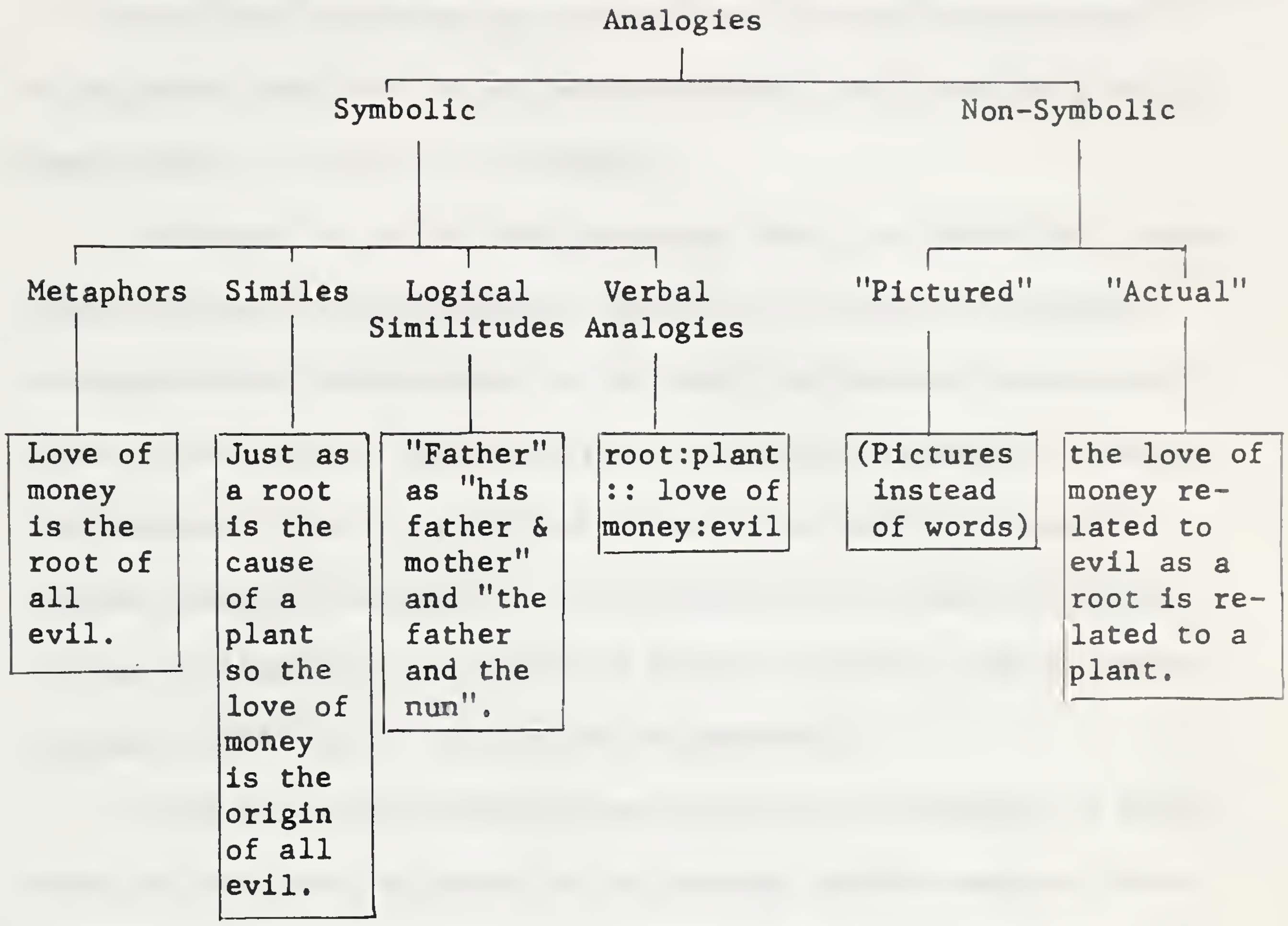


Figure 2. Classification of types of analogies.

> Just as certain types of exercise may increase the capacity and coordination of muscle tissue, so appropriate experience in analogical thinking develops the power to get meaning, and particularly that most difficult of all types of meaning involved in the adept solution of problems that are new and strange.[51]

He suggested that this is so because analogy makes complex abstract ideas easier to see and to remember.

Although it is not the intention, here, to report the literature regarding the philosophical aspect of a study of analogies, the unpublished dissertation of Harrison[52] affords an opportunity to explore some aspects of the subject. Harrison attempted to explore the problem, "Does it seem likely that there could be a useful general theory of analogy?" He discussed the use made of analogy and the writings on the subject by Plato, Aristotle, Saint Thomas Aquinas, Thomas de Vio Cajetan and Wittgenstein.

Both Plato and Aristotle made great use of analogy in their works and have made allusion to it although neither seems to have attempted a general theory. St. Thomas Aquinas, too, made use of analogy and mentions the subject often but offers no general theory.

The 1498 treatise of Cajetan[53] attempted to reduce all particular cases of analogy to three modes of analogy--analogy of inequality, analogy of attribution and analogy of proportionality. A proposed general theory was based on these modes of analogy.

---

[51] Ibid.

[52] Frank Russell Harrison, III, "Concerning the Possibility of a General Theory of Analogy" (unpublished Doctoral dissertation, The University of Virginia, 1961).

[53] Thomas de Vio Cajetan, The Analogy of Names and the Concept of Being (translator, Edward Buchinski) (Pittsburgh: Duquesne Press, 1959).

Wittgenstein[54] suggested, by referring to particular cases of analogy, that how we use our language and how we learn to master techniques within our language is more important than appealing to a general theory of analogy.

In the light of the study of the writings of the philosophers, Harrison suggested that there can be no general theory of analogy. To understand any particular analogy one has to turn to the particular case of that analogy. He recognized that one might suspect the possibility of a general theory because the general structure can so easily be expressed as A:B::C:D. Secondly, an analogy itself is meant to express some relation or ordering between its terms. However, it must be noted that we cannot specify any ordering or relation until the concrete terms of the particular analogy are mentioned. There may be several ways of viewing an analogy depending on the context or background (for example, grammatical, logical, biological, mathematical and so on).

We conclude then that the philosopher recognized the general form of an analogy as A:B::C:D but is unable to link the form to a general theory because the particular context of each analogy is vital to the recognition of the relationship. It is left to the realm of mathematics to accept a set of definitions which will allow the mathematician to manipulate the symbols which represent mathematical analogies (ratios). Insofar as the definitions are adhered to, many different particular problems may be solved through the use of the ratio concept.

[54]Ludwig Wittgenstein, Philosophical Investigations (translator, G. E. M. Anscombe) (Oxford: Basil Blackwell, 1958).

IV. SUMMARY

In compiling the first section of the review of related literature an attempt has been made to provide a background of information concerning concept formation, reasoning and problem solving abilities of children, based on an understanding of the way in which children think and learn. One would expect a detailed account of the work of Piaget. However, this report has been more concerned with the way in which the ideas expressed by Piaget have been related to the educational situation by such persons as Judd, LaZerte, Dienes, Isaacs and Van Engen.

The manner in which the research has been translated into practice in the teaching of the ratio concept is the subject of the second section. The teaching sequence proceeds from manipulative activity in grades two to five to an attempt to use strict mathematical notation to express the ratio concept.

The compilation of the third section presented special difficulties in that few investigators have chosen to discuss analogy in its own right. It was necessary to extract brief references from works primarily concerned with measuring mental capacity or with the development of language and thinking abilities. It is therefore difficult to express any conclusion which might be drawn from the review of the literature other than that writers interested in mental development and acquisition of language have often referred to the value of the process of analogical thinking. The same authors have often alluded to the use of analogy in mathematics, i.e. the ratio concept.

The next chapter reports the design of the experiment through which it was hoped to investigate the relationship between the ability to complete verbal analogies and an understanding of the ratio concept.

CHAPTER III

EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

Does the teaching of the ratio concept to children of grades five, six and seven influence their ability to choose the correct response to test items of the verbal analogies type? The main problem to be investigated in this study may be summarized by the above question.

The present chapter includes a description of the design of the experiment, a review of the testing equipment, a description of statistical procedures to be employed and a summary of the hypotheses.

I. SELECTION OF THE SAMPLE

The investigation was conducted in thirty-two grade seven classrooms in the Edmonton Public School system. Children in fifteen of the classrooms were using the _Seeing Through Mathematics_, Book One text, Unit Four of which attempts to teach the ratio concept as outlined in the previous chapter of this report. Children in the other seventeen classrooms were using the traditional textbooks, _Winston Mathematics, Intermediate Book I_[1] or _Arithmetic We Need Book 7_[2], in which ratio is given little or no emphasis. The

---

[1] W. L. Stein et al, _Winston Mathematics, Intermediate Book I_ (Toronto: John C. Winston Company, Limited, 1957).

[2] G. T. Buswell et al, _Arithmetic We Need, Book 7_ (Toronto: Ginn and Company, 1957).

seven schools involved represent all sections of the city and serve a population of varied economic circumstances.

It was possible to determine the history of instruction in arithmetic of these students through study of the cumulative record cards and/or personal contact with principals and teachers. The group included students who have had instruction in ratio according to the following schedule:

A. grades five, six and seven

B. grades six and seven

C grades five and six

D. grade seven

E. no formal instruction in ratio.

Also available from the cumulative record cards were the scores from the Laycock Mental Abilities Test which had been administered in the grade five level before the students had encountered instruction in ratio.

From the group of nine hundred thirty students, a randomized sampling of two hundred twenty-five were chosen to complete the sample as indicated in Table I. Simply stated, the sample consisted of fifteen measures in each of three intelligence groups and each of five treatment groups. In choosing the sample, it was necessary to eliminate students for whom all necessary information was not available and whose history of instruction in arithmetic did not fit any of the treatment groups of concern to the study. To this extent the sampling was not completely random. Figure 3 represents the top portion of the sheet designed to record the information needed to choose the sample. It will serve to summarize this section.

TABLE I

DATA PERTAINING TO THE SAMPLE

	HIGH INTELLIGENCE (More than 124)			AV. INTELLIGENCE (106 - 124)			LOW INTELLIGENCE (Less than 106)		
GROUP	Boys	Girls	Mean I.Q.	Boys	Girls	Mean I.Q.	Boys	Girls	Mean I.Q.
A	8	7	131	5	10	116	8	7	98
B	9	6	132	11	4	115	10	5	99
C	7	8	130	9	6	114	7	8	94
D	9	6	131	11	4	117	6	9	101
E	10	5	133	7	8	116	7	8	96

Name	Grade 5 and 6 School	Grade 5 Math	Grade 6 Math	Grade 7 Math Ratio (R) or No Ratio (N)	Treatment (Laycock- H, M or L) (Ratio - A, B, C, D or E)	Laycock Score	Student Number	Random Order	A.P.T. Score
Mary Brown	V. Park	R	R	N	C M	F 9	1	123	36
Bill White	Bellevue	N	R	R	B M	J 3	2	590	48
George Blue	Athlone	N	N	R	E –	–	3	71	17

Figure 3. Sheet used to record information needed in selection of the sample.

To avoid disruption of classes as much as possible the instrument was administered to all the students by the classroom teachers. To insure reasonable standardization in administration a special instruction sheet was given to, and discussed with, each teacher (Appendix A). Tests were given during the first week of June, 1964. Answer sheets were not corrected until after the random sampling was completed.

II. THE TESTING INSTRUMENT

In choosing a suitable testing instrument the following criteria were necessarily considered:

1) provision for the likelihood of a suitable range of test scores at grade seven level.
2) freedom from difficulties caused by lack of reading ability.
3) presence of items representing a variety of subject matter.
4) presence of items representing varying types of analogous relations.
5) dominance of power over speed as a factor in test administration.

Most intelligence and ability tests include verbal analogies. They may be grouped as a single sub-test or interspersed throughout the test. The Verbal section of the Academic Promise Test[3] appeared to best fit the criteria. The authors of the test make the following statement:

---

[3]George K. Bennet et al, Academic Promise Test (New York: The Psychological Corporation, 1962).

The design of the Academic Promise Test had specified that each test should provide a suitable range of scores throughout grades six through nine; should be primarily a power test, yet be short enough to permit administration in relatively little time; and should none-the-less be satisfactorily reliable within each single grade.[4]

The Verbal Section of the Academic Promise Test is composed of sixty items illustrated by the following example:

27. BARGAIN is to TRADE as MISLEAD is to

(a) Delude, (b) Purchase, (c) Advise, (d) Sell.[5]

The items are arranged in order of increasing difficulty and appear to meet the points of the criteria satisfactorily.

1) A study of the norms indicated that the mean score at the grade sevel level is twenty-nine items correct, with a standard deviation of 9.5 items.

2) Many of the test items include only words well within the reading vocabulary of grade seven students. The format of the test items stands as evidence that few service words are used, i.e., is, to, and as.

3) The concepts represented by the terms should be well within the understanding of grade seven students. Items represent a wide range of subject fields (for example, mathematics, biology, social studies, physiology, and geography, as well as many situations within the every-day experiences of children).

4) Test items represent a variety of relations. Three examples of completed items will serve to illustrate. (a) "paragraph is to

---

[4]George K. Bennet et al, Academic Promise Tests Manual (New York: The Psychological Corporation, 1962), p. 4.

[5]Ibid., p. 18.

sentence as sentence is to word" requires the recognition of a logical subtractive situation; (b) "tree is to fork as railroad is to branch" requires the comparison of a thing of nature to a man-made phenomenon; (c) "write is to teach as written is to taught" suggests a syntactic relation which must hold precedence over the meaning of the words.

5) Although a time limit of fifteen minutes is set for the test, the authors expected that this time limit would not be rigorous. In developing norms for the tests, the authors found that sixth grade pupils attempted at least three-quarters of the items while ninth graders attempted five-sixths of the items. (For the purposes of the study being reported no time limit was set.)

An important aspect of the choosing of a suitable instrument is the availability of statistical information regarding the test. The manual of the Academic Promise Test meets this criterion in that it includes lengthy reports on tests of validity, reliability, intercorrelation of the sub-tests and correlations with other tests. Because the test has been published for only a short time no reviews of the test were available at the time it was chosen for use in this study. The Sixth Mental Measurements Yearbook,[6] more recently published, substantiates the claims of the authors of the test. Stanley and Turnbull gave highly favorable reviews of the test in general and the verbal section in particular. Turnbull concluded that the analogies are well written and contain materials from the

---

[6] O. K. Buros (editor), The Sixth Mental Measurements Yearbook (Highland Park, N. J.: The Gryphon Press, 1965).

various disciplines as well as many every-day word relations. He cited the fact that the verbal scores correlate more highly with the scores in the numerical section than that of the language usage section as a strong indication that "the test appears to stress general verbal reasoning and general information rather than word relations more narrowly defined."[7]

A feature of the manual is the inclusion of the appropriate statistical evidence not only for the complete test but for each sub-test at each grade level. Statistics that are significant to the study being reported may be summarized in the following manner:

1) Correlation with course grades in English, Median r = .45;
2) Correlation with reading scores (grade six level in a city in north central United States) r = .54;
3) Correlation with course grades in mathematics, median r = .36;
4) Standard error of measurement, grade seven level, 3.6 points of raw score;
5) Reliability coefficient, r = .82;
6) Inter-correlation with numerical test, r = .69;
7) Inter-correlation with language usage test, r = .63.

One further aspect of the manual, which may be of interest to the study being reported, is the inclusion of tables which indicate the mean scores by grade and track in two cities of the United States. The purpose of the tables is to show that the Academic Promise Tests do, indeed, differentiate among groups of students

---

[7]Ibid., p. 767.

according to varying ability levels. In the case of a city in north central United States, the means for the verbal section is as follows:

Accelerated track	44.7
Fast track	38.0
Average track	29.5
Slow track	21.8

This information will be used in the discussion of the results of the study.

For purposes of separating students according to ability, scores of the _Laycock Mental Ability Test_, which were available from the cumulative record cards of the students, were used. The Laycock test has been used for many years by the Edmonton Public School system to gain an estimate of the intelligence of each child. Experience has shown that Edmonton students achieve relatively high scores on the test. This observation was substantiated when the scores for the population of this study were considered. It was necessary to consider scores of 106 to 124 as indicating average intelligence.

III. STATISTICAL PROCEDURES

In considering an experimental design it was necessary to consider one which could be fitted to a model suitable for use with the available computer. Analysis of covariance was necessarily used in order to remove potential sources of bias and to increase precision. In the case of the experiment being reported here, the independent variable or criterion measure was the score on the _Academic Promise Test_ and the covariate, the score on the _Laycock Mental Ability Test_.

Analysis of covariance yielded means for the variate free from bias produced by varying mean scores on the covariate. Stratification of the sample was simpler when it was unnecessary to obtain treatment samples of equal mean intelligence. Approximately fifteen observations were included in each treatment sample.

F-ratios obtained from the adjusted analysis of variance were considered in order to detect significant differences in means of the treatment samples. If an hypothesis was rejected, further analysis of the nature of the differences was made. Differences were considered significant if the obtained F-values were greater than F_{95}.

In cases where the hypothesis was rejected, a modified studentized range statistic, known as the Newman-Keuls method was used to explore the nature of the differences. The method was described fully by Winer.[8] Simply stated, the studentized range statistic is merely the difference between the largest and smallest treatment means divided by the square root of the mean square error over the number of observations.

$$\rho = \frac{\bar{T}\text{ largest} - \bar{T}\text{ smallest}}{\sqrt{\text{MSerror}/n}}$$

The use of the Newman-Keuls method made it possible to compare differences between all of the treatment means.

---

[8]B. J. Winer, Statistical Principles in Experimental Design (New York: McGraw-Hill Book Company, Inc., 1962).

IV. SUMMARY OF THE HYPOTHESES

As a matter of convenience the hypotheses are restated. It must be understood that each hypothesis was considered for the three levels of intelligence as indicated by the Laycock Mental Abilities Test scores. Thus, a total of twelve hypotheses were considered.

A) There is no difference in performance on the verbal analogies test between students who have had instruction in ratio in grades five, six and seven and those who have had no formal instruction in ratio.

B) There is no difference in performance on the verbal analogies test between students who have had instruction in ratio in grades six and seven and those who have had no formal instruction in ratio.

C) There is no difference in performance on a verbal analogies test between students who have had instruction in ratio in grades five and six and those who have had no formal instruction in ratio.

D) There is no difference in performance on a verbal analogies test between students who have had instruction in ratio in grade seven and those who have had no formal instruction in ratio.

Chapter IV reports the results of the statistical study and conclusions reached, as well as implications which may be drawn from a careful consideration of the results.

CHAPTER IV

RESULTS, CONCLUSIONS AND IMPLICATIONS

Findings of the investigation which was described in the previous chapter are presented here. Statistical evidence regarding mean scores on the verbal analogies test and conclusions drawn as a result of the evidence are given for each of the intelligence groups. Conclusions based on evidence not substantiated by statistical procedures are presented and discussed. Finally, implications which may be made in the light of the evidence but with consideration of the limitations of the study are presented.

I. STATISTICAL RESULTS

High Intelligence Group

Data pertaining to the high intelligence group is summarized in Table II. Analysis of covariance yielded F-ratios which indicated that differences in intelligence of the five groups were not significant at the .05 level nor were unadjusted or adjusted mean scores on the verbal analogies test significantly different at this level.

It may be concluded that there was no statistically significant difference in performance on a verbal analogies test among groups of students with high intelligence regardless of exposure to instruction in ratio.

TABLE II

COMPARISON OF HIGH INTELLIGENCE GROUPS RELATIVE TO MEAN SCORES ON VERBAL ANALOGIES TEST, LAYCOCK INTELLIGENCE AS COVARIANT

GROUP	MEAN INTELLIGENCE	UNADJUSTED MEAN, ANALOGIES TEST	ADJUSTED MEAN ANALOGIES TEST
A	131.2	41.400	41.523
B	132.2	43.533	43/327
C	130.3	42.400	42.830
D	130.8	44.333	44.588
E	133.4	44.800	44.199

Within Variance, Intelligence: F .85298 (d.f. 4/70)

Unadjusted Mean Scores, Analysis of Variance:
F 1.15385 (d.f. 4/70)

Adjusted Mean Scores, Analysis of Variance:
F .96062 (d.f. 4/69)

Average Intelligence Group

Data pertaining to the average intelligence group is summarized in Table III. Analysis of covariance yielded F-ratios which indicated that differences in intelligence of the five groups were not significant at the .05 level. However, differences in unadjusted and adjusted mean scores on the verbal analogies test were significant at this level.

It may be concluded that there was a statistically significant difference in performance on the verbal analogies test among groups of students of average intelligence depending upon exposure to instruction in ratio.

Further analysis of the findings using the Newman-Kuels method indicated significant differences when comparing treatment groups A and D with treatment group E. It may, therefore, be concluded that there was a statistically significant difference in performance on one verbal analogies test between students of average intelligence who had been exposed to instruction in ratio in grades five, six and seven and those who had not been exposed to formal instruction in ratio. A similar statement may be made when comparing students of average intelligence who had been exposed to instruction in ratio in grade seven only and those who had not been exposed to formal instruction in ratio.

TABLE III

COMPARISON OF AVERAGE INTELLIGENCE GROUPS RELATIVE TO MEAN SCORES ON VERBAL ANALOGIES TEST, LAYCOCK INTELLIGENCE AS COVARIANT

GROUP	MEAN INTELLIGENCE	UNADJUSTED MEAN ANALOGIES TEST	ADJUSTED MEAN ANALOGIES TEST
A	116.2	39.867	39.639
B	114.7	36.867	37.152
C	113.7	37.467	38.087
D	117.1	39.333	38.794
E	115.9	33.533	33.395

Within Variance, Intelligence: F .85056 (d.f. 4/70)

Unadjusted Mean Scores, Analysis of Variance:
F 2.88754 (d.f. 4/70)

Adjusted Mean Scores, Analysis of Variance:
F 3.00899 (d.f. 4/69)

NEWMAN-KEULS METHOD FOR DETERMINING SOURCE OF DIFFERENCES					
TREAT-MENTS	E	B	C	D	A
E		56.355	70.380	80.985*	93.660*
B			14.025	24.630	37.305
C				10.605	23.280
D					12.575

*Significant at .05 level.

<u>Low Intelligence Group</u>

Data pertaining to the low intelligence group is summarized in Table IV. Analysis of covariance yielded F-ratios which indicated that differences in intelligence of the five groups were not significant at the .05 level nor were unadjusted or adjusted mean scores on the verbal analogies test significantly different at this level.

It may be concluded that there was no statistically significant difference in performance on the verbal analogies test among students with low intelligence regardless of exposure to instruction in ratio.

In summary, the results of statistical tests provide evidence that grade seven students of average intelligence were able to achieve superior performance on the verbal analogies test if they had been exposed to instruction in ratio in grades five, six and seven. In particular, students who had been exposed to instruction in ratio during grades five, six and seven or during grade seven only were able to achieve significantly higher scores on the verbal analogies test. This conclusion is clearly illustrated in Figure 4. In contrast, instruction in ratio did not appear to have a significant effect on the ability of students of high or low intelligence to perform on the verbal analogies test.

TABLE IV

COMPARISON OF LOW INTELLIGENCE GROUPS RELATIVE TO MEAN SCORES ON VERBAL ANALOGIES TEST, LAYCOCK INTELLIGENCE AS COVARIANT

GROUP	MEAN INTELLIGENCE	UNADJUSTED MEAN, ANALOGIES TEST	ADJUSTED MEAN ANALOGIES TEST
A	97.93	30.600	30.459
B	99.0	32.133	31.628
C	94.5	27.533	28.576
D	100.5	33.667	32.637
E	95.7	29.800	30.433

Within Variance, Intelligence: $F = 2.46983$ (d.F. 4/70)

Unadjusted Mean Scores, Analysis of Variance:
$F = 2.23058$ (d.F. 4/70)

Adjusted Mean Scores, Analysis of Variance:
$F = .94501$ (d.F. 4/69)

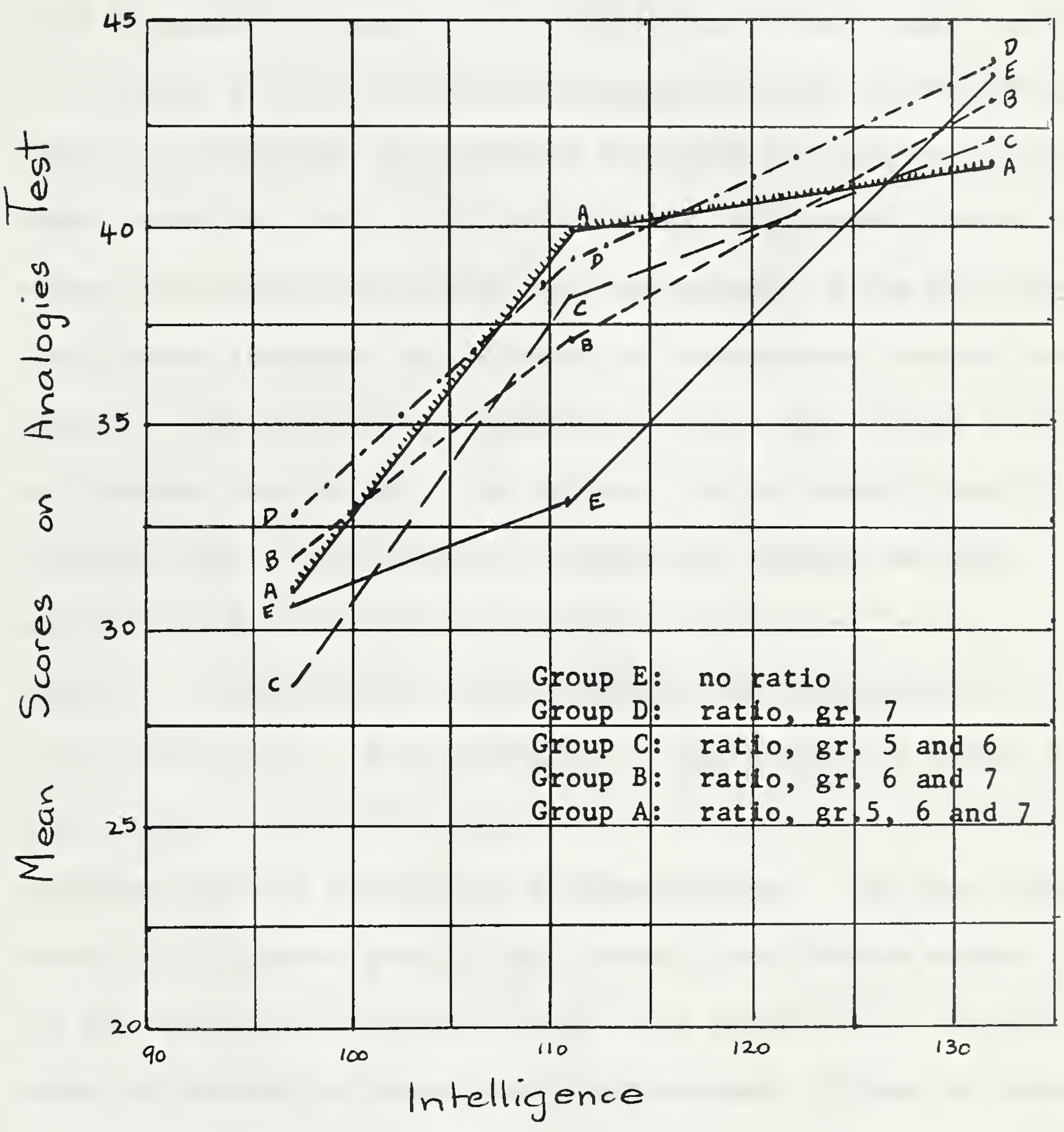


Figure 4. Mean scores on verbal analogies test compared with intelligence.

II. NON-STATISTICAL EVIDENCE

Evidence which is not verified by statistical procedures is summarized in Table V and illustrated in Figure 4 (page 67). Table V presents a comparison of differences between mean scores of low, average and high intelligence groups for each of the treatment groups. Differences are expressed in points of raw score on the verbal analogies test. It is evident that differences between mean scores on the verbal analogies test of students of low and average intelligence increased as the amount of instruction in ratio increased. This statement is particularly true when groups A and C are compared with group E. In contrast, it is evident that differences between mean scores of students of average and high intelligence decreased as the amount of instruction in ratio increased. This statement is particularly true when group A is compared with group E. This phenomenon is represented in Figure 4 by the slopes of the lines which join mean scores on the verbal analogies test for each of the treatment groups. The line joining scores for treatment group A has a steep slope between scores of low and average intelligence groups and a gradual slope between scores of average and high intelligence groups. Slopes of lines representing treatment groups C, B, D and E modify until the slope of the line representing the differences between low and average intelligence groups of treatment E is gradual and the slope which represents the difference between average and high intelligence groups is steep.

TABLE V

COMPARISON OF DIFFERENCES IN ADJUSTED MEAN SCORES ON THE VERBAL ANALOGIES TEST OF STUDENTS OF HIGH, AVERAGE AND LOW INTELLIGENCE

Sequence of instruction in ratio	Difference between mean scores of low and average intelligence groups	Difference between mean scores of average and high intelligence groups
Group E: none	3.0	10.8
Group D: grade 7 only	6.2	5.6
Group B: grades 6 and 7	5.5	6.2
Group C: grades 5 and 6	9.5	4.7
Group A: grades 5, 6 and 7	9.1	1.9

Note: Figures represent points of raw scores on the verbal analogies test.

In the light of the evidence it may be concluded that instruction in ratio at the grades five, six and seven levels makes it possible for the student of average intelligence to perform almost as well as the student of high intelligence on a verbal analogies test. In contrast, the student of average intelligence who has had little or no instruction in ratio does not perform much better on a verbal analogies test than the student of low intelligence.

Three aspects of the study warrant discussion:

1. There is statistically significant evidence that instruction in ratio makes it possible for the grade seven student of average intelligence to perform at a higher level on a verbal analogies test than the student who has been exposed to no instruction in ratio. Students of low and high intelligence do not appear to be affected in a similar manner.

2. There appears to be a relationship between the timing and amount of instruction in ratio and the effect of the instruction on the grade seven student of average ability, i.e., instruction in ratio in grade five and six has a more positive effect than instruction in ratio in grade six and seven.

3. There is evidence, which cannot be subjected to statistical tests at this time, that suggests that instruction in ratio makes it possible for average students to perform on a verbal analogies test at a level comparable to the performance of students of high intelligence. In contrast, students of average intelligence who have experienced no instruction in ratio show little more success on a verbal analogies test than students with low intelligence.

Why didn't instruction in ratio have a positive effect on students' ability to perform on the verbal analogies tests at all intelligence levels? There is evidence in the review of literature to explain this phenomenon. Research indicated that students of grades five, six and seven may be at a critical stage in the development of reasoning ability. Piaget suggested that children move from the stage of concrete operations to the stage of formal reasoning at age eleven, twelve or thirteen. If mental age rather than chronological age is considered it could be expected that most grade seven students of high intelligence would have reached the stage of formal reasoning and would be able to perform the higher reasoning tasks represented by verbal analogies. Other grade seven students may have been experiencing a period of transition between the stage of concrete operations and that of formal reasoning. Results of the study reported in this thesis indicate that the transitional stage may have been accelerated by the study of the ratio concept in the case of students of average intelligence. Therefore, they were able to achieve a higher performance on the verbal analogies test. Students of low intelligence did not appear to have experienced the same acceleration of the transitional stage.

If the suppositions outlined in the previous paragraph are valid, successful exposure to the ratio concept would demand introduction of the ratio concept through experiences at the concrete level with gradually increasing amounts of instruction of a formal nature. It is the contention of this writer that the methods prescribed in the <u>Seeing Through Arithmetic</u> texts follow the necessary

sequence of instruction. Grade five students are encouraged to manipulate groups of objects representing situations which illustrate the concept of ratio. Grade six students are expected to use the ratio test to solve problems which may be symbolized by pairs of ratios. Grade seven students learn how to express the ratio concept in strict mathematical notation. In the light of research evidence as to how children develop concepts and learn to use them it would be expected that omission of the initial stage of instruction would threaten the success of later instruction. Thus, instruction in ratio in grades five and six would be more effective than instruction in grades six and seven if provision were not made for experiences of a concrete nature.

The suggestion that instruction in ratio makes it possible for students of average intelligence to perform almost as well on a verbal analogies test as students of high intelligence may be regarded as evidence that both groups of students have reached the stage of formal reasoning. This conclusion is supported by hypotheses stated by Peel and Watts that progress towards the stage of formal reasoning may be accelerated through the use of carefully selected experiences in mathematics. The similarity between the format of proportional relations and verbal analogies suggests that instruction in ratio would provide ideal experiences.

III. LIMITATIONS AND IMPLICATIONS

The results of the experiment suggest the conclusion that study of ratio in mathematics makes it possible for children of average ability to achieve a higher performance on a verbal analogies test. Carried to broader application the conclusion suggests that study of ratio may tend to accelerate the development of reasoning ability. In the light of such an implication, limitations of the study and suggestions for further research are suggested in the final section of the thesis.

Of primary consideration in planning further study are the limitations of the study reported here. First, the validity of the grouping as determined by scores on the Laycock test must be questioned. The level of scores of students judged to be of average ability was unusually high. Does this indicate superior ability on the part of the population studied or inadequacies in the testing instrument? Second, the factors of reading skill and vocabulary development must be considered of significance in such a study. How did these factors influence the scores, especially of children of low ability? Third, the basis of the hypotheses of the study was, merely, the exposure to the ratio concept. Did children of low ability have sufficient grasp of the ratio concept to be able to use the skill in solving verbal analogies? This limitation is particularly significant because the teaching of a new program such as <u>Seeing Through Arithmetic</u> beginning at as high a level as grade five entails much preparation on the part of the teacher and considerable

re-learning on the part of the students. It might be said that teachers tend to teach to the "average ability child" as they familiarize themselves with new programs.

Replication of the study with careful consideration to the limitations should be undertaken. Control of the factor of mental age should be carefully considered in the light of Piagetian theories. Questions which demand answers are: (1) At what age are highly intelligent children capable of success in tasks of the verbal analogies type? (2) Will instruction which makes it possible for children of low intelligence to understand the ratio concept make it possible for them to gain success in tasks of the verbal analogies type? (3) Is the strict mathematical symbolization of the ratio concept, as taught at the grade seven level, necessary for the understanding of the concept?

With due consideration for the limitations of the study, it seems proper to suggest that educationists should consider the possibility that instruction in ratio may accelerate the development of reasoning ability. Programs of study and methods of teaching should be adapted so that any gain in reasoning ability may be capitalized on at an early age. If children in the upper elementary grades are able to use formal reasoning techniques they should be given greater opportunity to use and develop the technique. It is through development of this ability that the central purpose of education is most likely to be achieved.

BIBLIOGRAPHY

BIBLIOGRAPHY

Bennet, George K. et al. Academic Promise Tests. New York: The Psychological Corporation, 1962.

Bennet, George K. et al. Academic Promise Tests Manual. New York: The Psychological Corporation, 1962.

Brown, Joseph C. and Lotus D. Coffman. How To Teach Arithmetic. Chicago: Row, Peterson and Company, 1914.

Bruner, J. S. The Process of Education. Cambridge: Harvard University Press, 1960.

Buros, Oscar Krisen (editor). The Sixth Mental Measurements Yearbook. Highland Park: The Gryphon Press, 1965.

Burt, Cyril. "Experimental Tests of Higher Mental Processes and Their Relation to General Intelligence," Journal of Experimental Pedagogy, I (November, 1911), 94 - 112.

Buswell, Guy T. et al. Arithmetic We Need, Grade Seven. Toronto: Ginn and Company, 1955.

de Vio Cajetan, Thomas. The Analogy of Names and the Concept of Being. Trans. Edward Bushinski. Pittsburgh: Duquesne Press, 1959.

Dienes, Z. P. An Experimental Study of Mathematical Learning. London: Hutchenson and Company, Limited, 1963.

Dienes, Z. P. "On Learning of Mathematics," Arithmetic Teacher, 10 (March, 1963), 115 - 126.

Harrison, Frank Russell III. "Concerning the Possibility of a General Theory of Analogy." Unpublished Doctoral Dissertation, The University of Virginia, 1961.

Hartung, Maurice L. "Fractions and Related Symbolism in Elementary School Instruction," Elementary School Journal, LVIII (April, 1957), 377 - 384.

Hartung, Maurice L. et al. Charting the Course in Arithmetic. Chicago: Scott, Foresman and Company, 1960.

Hartung, Maurice L. et al. Numbers in Action. Toronto: W. J. Gage, Limited.

Hartung, Maurice L. et al. Numbers We See. Toronto: W. J. Gage, Limited.

Hartung, Maurice L. et al. Seeing Through Arithmetic. Chicago: Scott Foresman and Company, 1957.

Hartung, Maurice L. et al. Seeing Through Mathematics, Book I. Toronto: W. J. Gage, Limited.

Hartung, Maurice L. et al. Teachers' Guide, Seeing Through Arithmetic. Toronto: W. J. Gage, Limited.

Heidbreder, E. "The Attainment of Concepts: I, Terminology and Methodology." The Journal of General Psychology, 35 (1946), 173 - 189.

Isaacs, J. Nathan. The Growth of Understanding in the Young Child. London: The Educational Supply Association, 1961.

Judd, Charles H. Psychological Analysis of the Fundamentals of Arithmetic. Chicago: University of Chicago Press, 1927.

Knight, F. B. et al. Study Arithmetic, Book Six. Toronto: W. J. Gage, Limited.

LaZerte, Milton Ezra. The Development of Problem Solving Ability in Arithmetic, A Summary of Investigations. Toronto: Clarke, Irwin and Company, Limited, 1933.

Lindstedt, Sidney A. "Changes in Patterns of Thinking Produced by a Special Problem Solving Approach in Elementary Arithmetic." Unpublished Doctoral Dissertation, The University of Wisconsin, 1962.

National Education Association, Educational Policies Commission. The Central Purpose of American Education. Washington: National Education Association, 1961.

Peel, E. A. The Pupil's Thinking. London: Oldbourne Book Company, Limited, 1960.

Piaget, Jean. Judgment and Reasoning in the Child. New York: Harcourt, Brace and Company, 1928.

Raven, J. C. Progressive Matrices. London: H. K. Lewis and Company, 1956.

Spearman, C. The Abilities of Man. London: Macmillan and Company, 1932.

Stein, W. L. et al. Winston Mathematics, Intermediate Book One. Toronto: John C. Winston Company, Limited, 1952.

Trimble, H. C. "Fractions Are Ratios Too," Elementary School Journal, XLIX (March, 1949), 285 - 291.

Upton, Albert. Design for Thinking. Stanford: Stanford University Press, 1961.

Upton, Albert and Richard W. Samson. Creative Analysis. Whittier: Whittier College Press, 1961.

Van Engen, Henry. "Child's Introduction to Arithmetic Reasoning," School Science and Mathematics, 55 (May, 1963), 358 - 363.

Van Engen, Henry. "The Formation of Concepts", The Learning of Mathematics, Its Theory and Practice, pp. 69 - 92. Washington: National Council of the Teachers of Mathematics, 1953.

Van Engen, Henry. "Rate Pairs, Fractions and Rational Numbers," Arithmetic Teacher, 7(December, 1960), 389 - 399.

Van Engen, Henry and E. Glenadine Gibb. "Structuring of Arithmetic," Instruction in Arithmetic. Washington: National Council of the Teachers of Mathematics, 1960, pp. 31 - 61.

Van Engen, Henry et al. Teaching Guide, Seeing Through Mathematics, Book One. Chicago: Scott Foresman and Company, 1963.

Watts, A. F. The Language and Mental Development of Children. London: George C. Harrop and Company, Limited, 1944.

Wittgenstein, Ludwig. Philosophical Investigations. Trans. G. E. M. Anscombe. Oxford: Basil Blackwell, 1958.

APPENDIX

VERBAL SECTION OF THE ACADEMIC PROMISE TEST

DIRECTIONS TO THE TEACHER

ONLY THE VERBAL SECTION, PAGES 17, 18, and 19, IS TO BE GIVEN. INSTRUCTIONS HAVE BEEN CHANGED INSOFAR AS THE STUDENTS ARE TO BE GIVEN AS MUCH TIME AS IS NEEDED TO COMPLETE THE TEST. (Very slow workers need not complete the whole test.) The instructions which follow should be strictly adhered to.

1. Distribute answer sheets and give the class time to complete the information requested.

2. Distribute the test booklets, and request that they be opened to page 17. REMIND THE STUDENTS THAT THEY MUST NOT PUT ANY MARKS ON THE TEST BOOKLETS.

3. Say to the class, "READ THE DIRECTIONS FOR THE VERBAL TEST SILENTLY AS I READ THEM ALOUD."

   Pause after examples "S" and "T" for the students to mark their answers on the answer sheet.

4. When you have read the correct answers aloud, allow time for those who answered incorrectly to look at the items again, saying "CORRECT YOUR ANSWERS IF EITHER WAS WRONG."

5. Ask "ARE THERE ANY QUESTIONS?" Pause to answer questions. Answer by rereading appropriate sections of the directions or reviewing the examples given. DO NOT EMPLOY NEW EXPLANATIONS OR EXAMPLES.

6. Draw attention to the last paragraph of the directions saying, "YOU MAY HAVE AS MUCH TIME AS YOU NEED TO ANSWER ALL THE QUESTIONS. WORK QUICKLY BUT CAREFULLY. IT SHOULD TAKE ABOUT FIFTEEN MINUTES."

   Collect answers and tests after all but the slowest pupils have finished.

FORM **A**

apt

ABSTRACT REASONING
NUMERICAL
VERBAL
LANGUAGE USAGE

DIRECTIONS

Do not open this booklet until you are told to do so.

On Side 1 of the SEPARATE ANSWER SHEET, print your name, school and other requested information in the proper spaces. Blacken the spaces that indicate your sex, your grade, and the Form of the test.

This is **Form A.**

Wait for further instructions.

DO NOT MAKE ANY MARKS IN THIS BOOKLET.

The Psychological Corporation
304 East 45th Street
New York 17, New York

Printed in U.S.A.

61-136 TB

Do not make any marks in this booklet

VERBAL

Mark your answers on the separate Answer Sheet

DIRECTIONS

This test shows how well you understand the relationship between words. Look at Example R below:

Example **R.** LIGHT is to DARK as PLEASURE is to
(A) Picnic, (B) Day, (C) Pain, (D) Night

(C) Pain is the right answer because pain is the opposite of pleasure just as dark is the opposite of light. PAIN is related to PLEASURE in the same way that DARK is related to LIGHT.

In each line on the following pages, the first two words in capital letters are related in some way. You are to find what the relation is between those two words. Then you are to pick out a word which is related in the same way to the third word in capitals, and which will make the sentence true and sensible. Then on the answer sheet, fill in the space under the letter of the word you choose. For Example R, above, you would fill in the space under **C,** like this:

Sample of Answer Sheet

	A	B	C	D
R.			▌	

Now look at the following examples. Decide what the correct answers are, and fill in the spaces for the examples on your answer sheet.

Example **S.** CHEESE is to MILK as FLOUR is to
(A) Beef, (B) Beets, (C) Wheat, (D) Grapes

Example **T.** $1 is to DOLLAR as 1° is to
(A) Inch, (B) Quart, (C) Shilling, (D) Degree

The correct answers to Examples S and T are **C** and **D.**

You will have 15 minutes for this test. Work as rapidly and as accurately as you can. If you are not sure of an answer, mark the choice which is your best guess.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. NET is to FISHERMAN as WEB is to (A) Ant, (B) Fly, (C) Spider, (D) Beetle
2. CATERPILLAR is to BUTTERFLY as TADPOLE is to (A) Polecat, (B) Frog, (C) Locust, (D) Lodgepole
3. TELEGRAPH is to MORSE as ELECTRIC LIGHT is to (A) Edison, (B) Ford, (C) Franklin, (D) Bell
4. MAYOR is to CITY as GOVERNOR is to (A) Company, (B) Town, (C) State, (D) Kingdom
5. MASON is to STONE as WELDER is to (A) Cloth, (B) Wood, (C) Glass, (D) Metal
6. Z is to A as END is to (A) Bottom, (B) Beginning, (C) Top, (D) Side
7. BROOM is to DUSTPAN as MOP is to (A) Duster, (B) Sponge, (C) Pail, (D) Towel
8. ACORN is to OAK as BULB is to (A) Rose, (B) Blueberry, (C) Tulip, (D) Maple
9. NORTH AMERICA is to SOUTH AMERICA as MISSISSIPPI is to (A) Ganges, (B) Danube, (C) Seine, (D) Amazon
10. MUSCLES is to MOVEMENT as STOMACH is to (A) Digestion, (B) Warmth, (C) Elimination, (D) Ventilation
11. HAMMER is to NAIL as WRENCH is to (A) Bit, (B) Chisel, (C) Vise, (D) Bolt
12. CITY is to VILLAGE as ARMY is to (A) Ship, (B) Squad, (C) Corporal, (D) Captain
13. PENALTY is to REWARD as UNCERTAINTY is to (A) Confidence, (B) Health, (C) Appetite, (D) Prison
14. GROW is to SHRINK as INCREASE is to (A) Diminish, (B) Swell, (C) Maintain, (D) Precede
15. LOCOMOTIVE is to FREIGHT CAR as TUG is to (A) Taxi, (B) Bridge, (C) Wharf, (D) Barge
16. BELL is to TELEPHONE as McCORMICK is to (A) Reaper, (B) Airplane, (C) Locomotive, (D) Submarine
17. ISLAND is to PENINSULA as ICELAND is to (A) Florida, (B) Illinois, (C) California, (D) New York
18. EMPTY is to VACANT as BOLTED is to (A) Filled, (B) Dark, (C) Locked, (D) Loose
19. BUY is to PERMANENT as RENT is to (A) Convenient, (B) Certain, (C) Difficult, (D) Temporary
20. RAIN is to MUD as DROUGHT is to (A) Wind, (B) Dust, (C) Ice, (D) Flood
21. BIRD is to FISH as AIRPLANE is to (A) Rocket, (B) Whale, (C) Balloon, (D) Submarine
22. PELT is to SQUIRREL as RIND is to (A) Orange, (B) Child, (C) Pet, (D) Wing
23. WORLD WAR I is to WORLD WAR II as WILSON is to (A) Lincoln, (B) Roosevelt, (C) Grant, (D) Eisenhower
24. CORN is to IOWA as COAL is to (A) Connecticut, (B) Pennsylvania, (C) South Carolina, (D) Oregon
25. THEFT is to JAIL as MUTINY is to (A) Decoration, (B) Probation, (C) Promotion, (D) Execution
26. LONDON is to THAMES as NEW ORLEANS is to (A) Louisiana, (B) Mississippi, (C) Mexico, (D) Missouri
27. BARGAIN is to TRADE as MISLEAD is to (A) Delude, (B) Purchase, (C) Advise, (D) Sell
28. AIRPLANE is to WRIGHT as WIRELESS is to (A) Marconi, (B) Eastman, (C) Whitney, (D) Galileo
29. INDIAN is to BOW as REDCOAT is to (A) Musket, (B) Bullet, (C) Spear, (D) Cannon
30. WRITE is to TEACH as WRITTEN is to (A) Teaching, (B) Understood, (C) Learned, (D) Taught

GO ON TO THE NEXT PAGE

31. BANKER is to EXPLORER as CAUTION is to (A) Wealth, (B) Tropics, (C) Daring, (D) Safari

32. SUM is to REMAINDER as PRODUCT is to (A) Fraction, (B) Total, (C) Quotient, (D) Decimal

33. FORMOSA is to ASIA as MADAGASCAR is to
(A) Europe, (B) Australia, (C) South America, (D) Africa

34. STARTLE is to FRIGHTEN as CHANGE is to (A) Prepare, (B) Alter, (C) Hesitate, (D) Retain

35. CANADA is to NORTH AMERICA as CHINA is to (A) India, (B) Australia, (C) Asia, (D) Africa

36. PARAGRAPH is to SENTENCE as SENTENCE is to (A) Thought, (B) Deed, (C) Letter, (D) Word

37. HOUSE is to NAILS as DRESS is to (A) Buttons, (B) Stitches, (C) Hooks, (D) Zippers

38. ACCUSE is to CHARGE as FREE is to (A) Give, (B) Liberate, (C) Declare, (D) Reward

39. THIEF is to THIEVES as BELIEF is to (A) Believes, (B) Beliefs, (C) Knowledge, (D) Thoughts

40. TV is to RADIO as VISUAL is to (A) Nasal, (B) Sensitive, (C) Aural, (D) Tactile

41. FEATHER is to HAT as EMBROIDERY is to (A) Blouse, (B) Wool, (C) Needle, (D) Hand

42. FRIENDSHIP is to HARMONY as HOSTILITY is to
(A) Sickness, (B) Acquaintance, (C) Expense, (D) Discord

43. UNITED STATES is to CANADA as STATE is to (A) District, (B) Shire, (C) Province, (D) Parish

44. RED is to BLUE as ORANGE is to (A) Black, (B) White, (C) Green, (D) Yellow

45. SOW is to PIG as EWE is to (A) Hog, (B) Lamb, (C) Kid, (D) Filly

46. GRANT is to LEE as WASHINGTON is to (A) Lafayette, (B) Cornwallis, (C) George III, (D) Howe

47. TAKE is to ACCEPT as SUPPORT is to (A) Give, (B) Believe, (C) Borrow, (D) Maintain

48. THREE is to SIX as TRIANGLE is to (A) Diamond, (B) Hexagon, (C) Trapezoid, (D) Octagon

49. BUTTER is to BREAD as SUGAR is to (A) Candy, (B) Taffy, (C) Molasses, (D) Oatmeal

50. COCONUT is to ACORN as PALM is to (A) Maple, (B) Spruce, (C) Oak, (D) Elm

51. MERCIFUL is to PITILESS as FORGIVING is to
(A) Generous, (B) Vindictive, (C) Wealthy, (D) Indulgent

52. JOHNSON is to LINCOLN as ARTHUR is to (A) Adams, (B) Tyler, (C) Wilson, (D) Garfield

53. PAINTING is to PHOTOGRAPH as NOVELIST is to (A) Poet, (B) Artist, (C) Reporter, (D) Vocalist

54. MOUNTAIN is to SWAMP as ALLEN is to (A) Wolfe, (B) Marion, (C) Burgoyne, (D) Sheridan

55. PLURAL is to SINGULAR as BACTERIA is to (A) Oddity, (B) Germs, (C) Bacterium, (D) Fungi

56. SQUARE is to RECTANGLE as CIRCLE is to (A) Torus, (B) Curve, (C) Traffic, (D) Ellipse

57. 1 is to 1000 as YEAR is to (A) Millenium, (B) Century, (C) Decade, (D) Epoch

58. WHEEL is to TILLER as CAR is to (A) Boat, (B) Farm, (C) Tanker, (D) Truck

59. TREE is to FORK as RAILROAD is to (A) Branch, (B) Track, (C) Switch, (D) Station

60. PINE is to BIRCH as EVERGREEN is to (A) Everlasting, (B) Deciduous, (C) Leafy, (D) Strong

OP HERE. YOU MAY CHECK YOUR WORK ON THIS TEST. DO NOT GO BACK TO ANY PREVIOUS TEST.
DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.